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# LQR Control of Double Inverted-Pendulum Based on Genetic Algorithm

**Jin xiaochen**

Higher Vocational and Technical College,
The Liaoning University of Science and Technology, Anshan, China
E-mail: 649303709@qq.com

***Abstract—In practice, the key problem to apply LQR optimal control method is how to correctly choose the weighted matrix of performance index. At present, there is no formulaic approach for this problem. To obtain the satisfying results, people must repeat to test many times. This kind of LQR control method based on genetic algorithms, which can obtain satisfying control results at first hand, is presented for double inverted pendulum system. The method optimizes the Q-matrix by using genetic algorithms, selects trace of the result of Riccati equation as the objective function. The control problem of double inverted pendulum is resolved successfully. The simulation results prove that the control effect by this method is better than the other methods mentioned in the references.***

*Keywords*—genetic algorithm, double inverted-pendulum, LQR

## 1.Introduction

LQR optimal control is a basic method of modern control theory, because of its optimal solution can be written as a unified expression, but also can reflect the control law of a simple state feedback, the calculation and engineering implementation are relatively easy, so the method for automatic control of engineers and the theory has a great attraction to [1-3]. But using the above theory to solve the optimal control is carried out under the condition that the state variables of the system and the weighting matrix of the input variables have been selected. The effect depends on the selection of the weighting matrix. If the selection of these weighted matrices is inappropriate, it is possible to get a completely meaningless solution, not to talk about the "optimal". In practice, the correct choice of weighting matrix is not easy, want to make they can correctly reflect the actual situation of the whole control process, often require repeated modification to obtain satisfactory results, this greatly limits the two optimum control theory application. Genetic algorithm is a new optimization algorithm, J.H.Holland is a professor of the University of Michigan in 60s founded the search algorithm of natural selection and genetics based on the principle of. What it needs
The amount of information is small, and the global optimization can be found by a simple calculation.
Solution. Therefore, genetic algorithm has been widely applied in many areas, such as function optimization [4], automatic control [5], artificial neural network and optimal scheduling. In the field of intelligent control, genetic algorithm has great potential and wide application prospect. In this paper, the control problem of two stage inverted pendulum is solved by combining the genetic algorithm with the LQR control algorithm, and the control problem of the two stage pendulum is successfully solved.

## 2. The mathematical model of two stage inverted pendulum

Ignoring the air flow and friction, the system can be abstracted into the inverted pendulum car, homogeneous rod and mass, the actual system parameters are as follows: two straight inverted pendulum M=1.32Kg (car quality); $m_1$ =0.04Kg(pendulum1); $l_1$ =0.09m(Center to centroid distance of swing center 1); $m_2$ =0.132Kg (2 quality of pendulum rod); $l_2$ =0.27m(Center to centroid distance of swing center 2); $m_3$ =0.208Kg; F(The external force on the system); $\theta_1$ (The angle between the pendulum 1 and the vertical upward direction)； $\theta_2$ （The angle between the pendulum 2 and the vertical upward direction）。

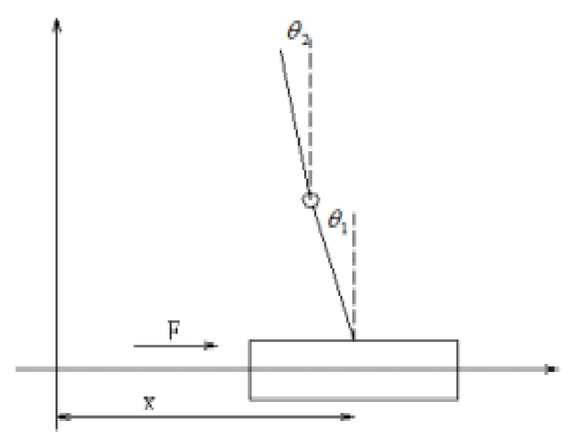


Figure 1 a schematic diagram of a two level inverted pendulum

2.1The mathematical model of two stage inverted pendulum

According to the reference and the inverted pendulum of Shenzhen Gu Gao company, the state space equation of the two stage inverted pendulum is established as follows:

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$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_{12} & K_{13} & 0 & 0 & 0 \\ 0 & K_{22} & K_{23} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ K_{17} \\ K_{27} \end{bmatrix} u \quad (1)$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u \quad (2)$$

## 3. Design of LQR controller based on genetic algorithm

Genetic algorithm optimization weights:

For the selection of matrix value, after simulating several times, the corresponding simulation results of the system are not very satisfactory. In view of this, we use genetic algorithm to optimize it. Because the weighted array can reflect the dynamic performance index of the system more often, in practical engineering applications, different array parameters are often chosen, and the array is selected as the unit matrix, and the weighted matrix is only optimized. The specific steps of optimization using genetic algorithms are as follows:

3.1 Determining the mathematical description of the objective function J in the LQR optimal control

$$J = \frac{1}{2}\int_{t_0}^{\infty}[X^T(t)Q(t)X(t) + u^T(t)R(t)u(t)]dt \quad (3)$$

Q is the parameter we need to optimize.

### 3.2 Optimization of Q matrix in LQR by genetic algorithm

3.2.1Chromosome coding method for determining parameters

The problem with binary encoding parameters constitute a substring, and then handle the string spliced into a chromosome string (string encoding length and form a great impact on the convergence of the algorithm). The purpose of this paper is to optimize $Q$ the selection matrix, so matrix diagonal 8 elements as the optimization parameters, the length of 10 bit binary encoding string to represent the 8 parameters respectively, and then the binary encoding string connected together to form a bit length binary code string; $R$ =1.

3.2.2Decode method

When decoding, we first need to cut the bit length binary encoding string into 8×10 bit long binary encoding strings, and then transform them into corresponding decimal integer codes[6].

3.2.3Design genetic operator

The genetic algorithm uses the selection operator to perform the survival of the fittest for the individual in the group. In the selection, the adaptability is the choice principle. The adaptability criterion embodies the natural law of the survival of the fittest and the obsolete. The probability of individuals with higher fitness to be inherited in the next generation population is larger, and the probability of being inherited to the next generation is smaller than that of the lower adaptive individuals. The most commonly used selection operator is the proportion selection operator in the basic genetic algorithm, which is used in this paper. The basic idea is that the probability of the individual being selected is proportional to the size of its fitness. Set the size of the group and the fitness of the individual, then the probability that the individual is selected is as follows:

$$Pi = \frac{Fi}{\sum_{i=1}^{M} Fi} \quad (4)$$

From the upper level, we can see that the number of the next generation is much more in the individuals with high fitness. Individuals with low adaptation are less and even eliminated in the next generation. In this way, the future generations have a strong ability to adapt to the environment. For the problem solving point of view, it is to choose the intermediate solution which is close to the optimal solution. In this paper, the selection operation is proportional selection operator, single point crossover operator is used in cross operation, and basic bit mutation operator is used in the mutation operation.

3.2.4Selecting control parameters of genetic algorithm

In order to ensure the search in the whole solution space, the initial population is randomly generated, the size of the population is 50, and the maximum iteration number is 100. When two individuals are selected to reproduce the next generation of individuals, random selection of the same position of two individuals, according to the cross probability p, is exchanged in the selected position. This process reflects the exchange of random information; the purpose is to produce a new combination of genes, that is, to produce new individuals. The crossover operator means that 2 pairs of paired chromosomes exchange part of their genes in a certain way, thus forming 2 new individuals. Cross operation is an important feature of genetic algorithm, which is different from other evolutionary algorithms. It plays a key role in genetic algorithm and is the main method to produce new individuals. In this paper, we use intermediate recombination double point crossover method, that is to say, we randomly select 2 individuals in the group, and only

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randomly set 2 crossover points in the individual coding string, and then exchange 2 partial chromosomes of the paired individuals with probability P=0.9 at this point.

According to the principle of genetic variation in biological inheritance, the mutation probability PM is applied to some individuals. In the case of variation, the inverse of the bit to be executed is to turn 1 into 0, and 0 to 1. The mutation probability PM is consistent with the case of biological variation minimum. Mutation operation is to replace the alleles of some loci in the chromosome coding strings of individuals with the other alleles of the loci and form a new individual. The mutation operation used in this paper is high variation, the mutation probability is selected as 0.01 and the initial population is generated randomly. Mutation can make the genetic algorithm have the function of local random search, and it can also maintain the diversity of groups and avoid initial convergence. That is to say, mutation increases the possibility of global optimization.

3.2.5Determination of individual evaluation method

In this design, the objective function is as follows:

$$J = trace(P) \tag{5}$$

Here is the solution of the Riccati equation (2.4).

$$(A-BK)^T P + P(A-BK) + K^T RK + Q = 0 \tag{6}$$

The fitness is the reciprocal of the target function value, that is:

$$f = \frac{1}{J} = \frac{1}{trace(P)} \tag{7}$$

3.2.6Genetic algorithm search process

The objective function by random sampling method, the selection of individuals through crossover and mutation to generate new individuals, then calculate the new individual value and the corresponding value of fitness function, the individual fitness function value with a generation to compare function value, the value of fitness function is more likely to larger genetic conditions generation[7].

3.2.7The condition of iterative termination of genetic algorithm

Theoretically, when the maximum of the fitness function is known or the lower bound of the fitness of the quasi optimal solution can be determined, we usually find the maximal or quasi optimal solution to be the iterative stopping condition of genetic algorithm. But when we implement genetic optimization, if we find that the evolution of individuals who occupy a certain proportion of the population has stabilized, that is to say, if we find that a certain proportion of the group of individuals is the same individuals, then we will terminate the algorithm iteration. In fact, when the genetic search is convergent, the fitness value of most of the individuals in the solution group is located near a certain value. This is used as the end condition of the algorithm. Or the number of iterations exceeds the given maximum generation, then the algorithm ends; otherwise, repeat [8].

Through the above algorithm, the value of the optimized element in the weighted matrix of the objective function is determined, and the vector of the feedback control law is determined. The MATLAB simulation program flow is as follows:

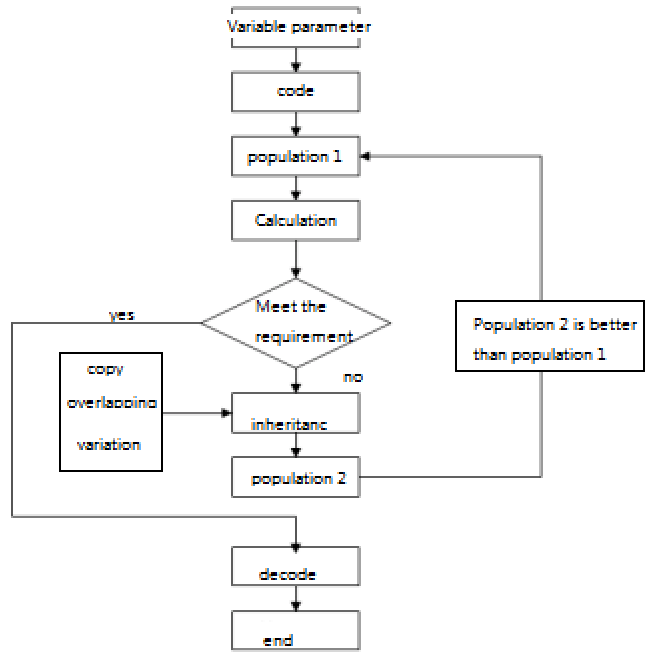


Figure 2 genetic algorithm flow

The genetic algorithm is optimized

$$Q = diag([419.5 \quad 61.5 \quad 197.1 \quad 931.6 \quad 614.2 \quad 305.2])$$

Using the optimized Q array, the control design and Simulation of the system are carried out, and the simulation curves of the output of the two stage inverted pendulum are obtained. These results are given in figure 3~ 6. The real line in the graph is the method in this paper. The dotted line is a method mentioned in reference in the reference. The selection of the matrix $Q$ in this document is $Q = C' \times C$ .

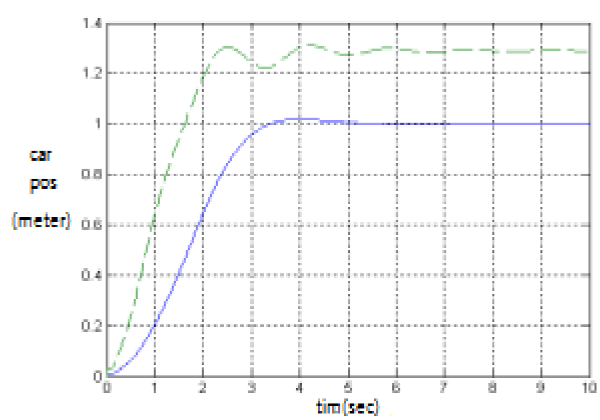


Figure 3 response curve of the trolley position system

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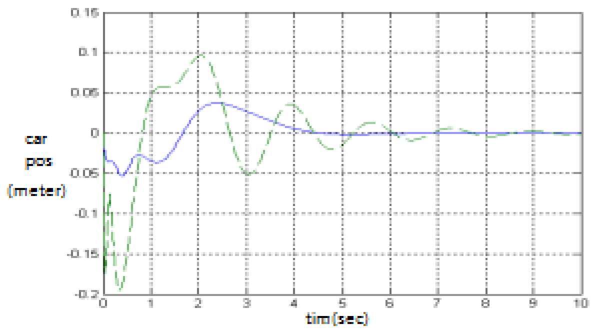


Figure 4 response curve of the first order pendulum system

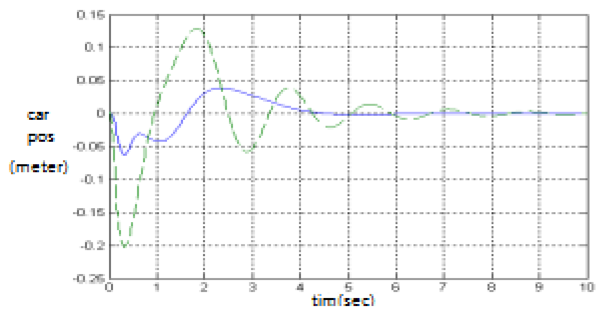


Figure 5 response curve of two stage pendulum system

As you can see from Figure 3, the result of the empirical formula can not reach the predetermined position, and the shock is more severe. Figure 4-5 is the system response curve of the two pendulum. From the vertical axis can be seen, whether the results obtained by genetic algorithm optimization Q array dynamic performance and static performance obviously than the response results behind the experience of Q array, and is mentioned in the literature two inverted pendulum of the left superior genetic algorithm, both the response speed of the system or overshoot are obvious to improve.

## 4. conclusion

Through the analysis of the above simulation results, we can see that this algorithm has good control effect on the control of multivariable, strong coupling and nonlinear system, and it can be used to control more complex objects, such as the two level inverted pendulum system. At the same time, we can see that the trace of the Riccati equation of the application system is a method of optimizing target by genetic algorithm, which can make the control effect of the system better and control accuracy higher. Therefore, this method has a good application prospect in complex system control.

## Reference

i. *M Senthil Kumar; S Vijayarangan. "Design of LQR controller for active suspension system". ISSN: 0971-4588. Indian journal of engineering and materials sciences. Vol.13, no.3. pp-173-179, 2006.*

ii. *Daniel E. Miller. "Near Optimal LQR Performance for a Compact Set of Plants". ISSN: 0018-9286. IEEE Transactions on Automatic Control. Vol.51, No9. 2006.*

iii. *Shun-Fa Hwang, Rong-Song He. "A hybrid real-parameter genetic algorithm for function optimization". ISSN: 1474-0346. Advanced Engineering Informatics. Vol.20, No.1. 2006.*

iv. *Hyun-Su Kim; Paul N. Roschke. "Design of fuzzy logic controller for smart base isolation system usinggenetic algorithm". ISSN: 0141-0296. Engineering Structures. Vol.28. No.1. 2006*

v. *Oliveira,Anabela; Baptista,Antonio M."On the role of tracking on Eulerian-Lagrangian solutions of the transport equation". Elsevier Sci Ltd, Exeter, Engl. Vol 21, no 7. p539-554, 1998.6.15.*

vi. *Holzhuter,Thomas. "Optimal regulator for the inverted pendulum via euler-Lagrange backward integration". Elsevier Ltd, Oxford, United Kingdom. Vol 40, no 9. p1613-1620, 2004.9.*

vii. *Song,Jun-Lie;Xiao,Jun;Xu,Xin-He. "Modeling and control method of the inverted pendulum system". Northeastern University. Vol 23, no 4. p333-337, 2002.4.*

viii. *Zhang,Li-Juan;Tu,Ya-Qing;Wang,Bao-Zhong. "Mathematical model of cart inverted pendulum system". Central South University of Technology,Changsha,China. Vol 36, no SUPPL. p199-202, 2005.8.*