Static Flexural Analysis of Thick Isotropic Beam Using Hyperbolic Shear Deformation Theory

P. M. Pankade, D. H. Tupe, S. B. Salve
Department of Civil Engineering, Deogiri Institute of Engineering and Management Studies, Aurangabad (M.S)-431005, India.
Email: durgeshtupe@gmail.com

Abstract-Flat slabs are highly versatile elements widely used in construction, providing minimum depth, fast construction and allowing flexible column grids. Common practice of design and In the present study, a hyperbolic shear deformation theory is developed for static flexural analysis of thick isotropic beams. The theory assumes a parabolic variation of transverse shear stress across the thickness of the beams. Simply supported thick isotropic beams analysed for the axial displacement, Transverse displacement, Axial bending stress and transverse shear stress. In this theory the hyperbolic sine and cosine function is used in the displacement field to represent the shear deformation e ect and satisfy the zero transverse shear stress condition at top and bottom surface of the beams. The Governing di erential equation and boundary conditions of the theory are obtained by using Principle of virtual work. The simply supported isotropic beam subjected to varying load is examined using present theory. The numerical results have been computed for various lengths to thickness ratios of the beams and the results obtained are compared with those of Elementary, Timoshenko, Trigonometric and other higher order refined theories and with the available solution in the literature

Keywords:
Thick beam, shear deformation, isotropic beam, transverse shear stress, static flexure, hyperbolic shear deformation theory, principle of virtual wor

The wide spreaduse of shear þexible materials in air craft, automotive, shipbuilding and other industries has stimulated interest in the accurate prediction of structuralbehaviourofbeams. Theories of beams involve basically the reduction of a three dimensional problems of elasticity theory to a one-dimensional problems. Since the thickness dimension is much smaller than the longitudinal dimension, it is possible to approximate the distribution of the displacement, strain and stress components in the thickness dimension. The various methods of development of reýned theories based on the reduction of the three dimensional problems of mechanics of elastic bodies are discussed by Gol denveizer [1], Kil chevskiy [2], Donnell [3], Vlasov and Leontev [4], Sayir and Mitropoulos [5].

It is well-known that elementary theory of bending of beam based on Euler-Bernoulli hypothesis that the plane sections which are perpendicular to the neutral layer before bending remain plane and perpendicular to the neutral layer after bending, implying that the transverse shear and transverse normal strains are zero. Thus the theory disregards the effects of the shear deformation. It is also known as classical beam theory. The theory is applicable to slender beams and should not be applied to thick or deep beams. When elementary theory of beam (ETB) is used for the analysis thick beams, deþections are underestimated and natural frequencies and buckling loads are overestimated. This is the consequence of neglecting transverse shear deformations in ETB. Rankine [6], Bresse [7] were the ýrst to include both the rotatory inertia and shear þexibility e ects as reýned dynamical e ects in beam theory. This theory is, however, referred to as the Timoshenko beam theory as mentioned in the literature by Rebello, et al. [8] and based upon kinematics it is known as ýrst-order shear deformation theory (FSDT). Rayleigh [9] included the rotator inertia effect while later the effect of shear stiffness was added by Timoshenko [10]. Timoshenko showed that the e ect of shear is much greater than that of rotatory inertia for transverse vibration of prismatic beams. In Timoshenko beam theory transverse shear strain distribution is constant through the beam thickness and therefore requires shear correction factor to correct the strain energy of deformation. Cowper [11] and Murty [13] have given new expressions for this coe cient for di erent cross-sections of the beam. Stephen and Levinson [15] have introduced a reýned theory incorporating shear curvature, transverse direct stress and rotatory inertia e ects. The limitations of the elementary theory of bending (ETB) of beams and ýrst order shear deformation theory (FSDT) for beams forced the development of higher order shear deformation theories Soler [16] developed the higher order theory for thick isotropic rectangular elastic beams using Legendre polynomials and Tsai and Soler [17] extended it to orthotropic beams. E ects of shear deformation and transverse normal stress are included. Levinson [18] obtained the higher order beam theory providing the fourth order system of di erential equations, satisfying two boundary conditions at each end of the beam. No shear correction factors are required since the theory satisýes the shear stress free surface conditions on the top and bottom of the beam. Krishna Murty [22] formulated a third order beam theory including the transverse shear strain and non classical (nonlinear) axial stress. In this theory the parabolic transverse shear stress distribution across the depth of the beam can be obtained using constitutive relations. Ghugal and Dahake[23] has developed a trigonometric shear deformation theory for þexure of thick or deep beams, taking into account transverse shear deformation e ect. The number of variables in the present theory is same as that in the

ISSN:2319-6890(online),2347-5013(print)
27-28 Feb. 2016

ýrst order shear deformation theory. The sinusoidal function is used in displacement ýeld in terms of thickness coordinate to represent the shear deformation e ects. This theory obviates the need of shear correction factor. Ghugal and Sharma [26], Sayyad and Ghugal [28] developed a variationally consistent reýned hyperbolic shear deformation theory for þexure and free vibration of thick isotropic beam. This theory takes into account transverse shear deformations e ects. In this paper, a hyperbolic shear deformation theory is developed for static þexural analysis of thick isotropic beams. The theory is applied to a Simply supported thick isotropic beams to analysed the axial displacement, Transverse displacement, Axial bending stress and transverse shear stress. The numerical results obtained for various lengths to thickness ratios of the beams and the results obtained are compared with those of Elementary, Timoshenko, Trigonometric and other higher order rreýned theories and with the available solution in the literature.

2.Formulation of Problem

Consider a thick isotropic simply supported beam of length L in x direction, Width b in y direction and depth h as shown in Figure 1. Where x, y, z are Cartesian coordinates. The beam is subjected to transverse load of intensity q (x) per unit length of beam. Under this condition, the axial displacement, Transverse displacement, Axial bending stress and transverse shear stress are required to be determined.

2.2 Assumptions made in the theoretical formulation:

1. The axial displacement (u) consist of two parts:
(a) Displacement given by elementary theory of bending.
(b) Displacement due to shear deformation, which is assumed to be hyperobolic in nature with respect to thickness coordinate.
2. The transverse displacement (w) in z direction is assumed to be function of x coordinate.
3. One dimensional constitutive laws are used.
4. The beam is subjected to lateral load only.

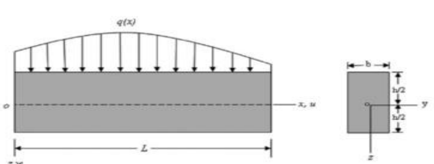

Figure 1: Simply supported beam bending under x-z Plane

2.3 The Displacement ýeld:

Based on the above mentioned assumptions, the displacement ýeld of the present beam theory can be expressed as follows. The hyperbolic function is assigned according to the shearing stress distribution
through the thickness of beam.

Where

$$u(x,z,t) = -z\frac{\partial w}{\partial x}(x,t) + \left[h\sinh\left(\frac{z}{h}\right) - \frac{4}{3}\frac{z^3}{h^2}\cosh\left(\frac{1}{2}\right)\right]\phi(x,t) \quad (1)$$

$$w(x,t) = w(x,t) \quad (2)$$

u = Axial displacement in x direction which is function of x, z and t.
w = Transverse displacement in z direction which is function of x and t.
ű=Rotation of cross section of beam at neutral axis

due to shear which is an unknown function to be determined and it is a function of x and t.
Normal strain:

$$\epsilon_x = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} + \left[h\sinh\left(\frac{z}{h}\right) - \frac{4}{3}\frac{z^3}{h^2}\cosh\left(\frac{1}{2}\right)\right]\frac{\partial \phi}{\partial x} \quad (3)$$

Shear Strain

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \left[\cosh\left(\frac{z}{h}\right) - 4\frac{z^2}{h^2}\cosh\left(\frac{1}{2}\right)\right]\phi \quad (4)$$

Stresses:

The one dimensional Hooke's law is applied for isotropic material, stress x is related to strain x and shear stress is related to shear strain by the following constitutive relations.

$$\sigma_x = E\epsilon_x = -zE\frac{\partial^2 w}{\partial x^2} + E\left[h\sinh\left(\frac{z}{h}\right) - \frac{4}{3}\frac{z^3}{h^2}\cosh\left(\frac{1}{2}\right)\right]\frac{\partial \phi}{\partial x} \quad (5)$$

$$\tau_{xz} = G\gamma_{xz} = G\left[\cosh\left(\frac{z}{h}\right) - 4\frac{z^2}{h^2}\cosh\left(\frac{1}{2}\right)\right]\phi \quad (6)$$

where E and G are the elastic constants of the beam material.

2.4 Governing differential equations:

Governing di erential equations and boundary conditions are obtained from Principle of virtual work. Using equations for stresses, strains and principle of virtual work, variationally consistent differential equations for beam under consideration are obtained. The principle of virtual work when applied to beam leads to:

ISSN:2319-6890(online),2347-5013(print)
27-28 Feb. 2016

$$b\int_{x=0}^{x=L}\int_{z=-h/2}^{z=h/2}(\sigma_x\cdot\delta\epsilon_x+\tau_{xz}\cdot\delta\gamma_{xz})\,dxdz + \rho b\int_{x=0}^{x=L}\int_{z=-h/2}^{z=h/2}\left(\frac{\partial^2 u}{\partial t^2}\cdot\delta u+\frac{\partial^2 w}{\partial t^2}\cdot\delta w\right)dxdz - \int_{x=0}^{x=L} q\delta w dx = 0 \quad (7)$$

Employing Greens theorem in above Equation successively, we obtain the coupled Euler-Lagrange equations which are the governing differential equations and associated boundary conditions of the beam. The governing differential equations obtained are as follows:

$$EI\left[\frac{\partial^4 w}{\partial x^4}-A_0\frac{\partial^3\phi}{\partial x^3}\right]-\rho I\left[\frac{\partial^4 w}{\partial x^2\partial t^2}-A_0\frac{\partial^3\phi}{\partial x\partial t^2}\right]+\rho A\frac{\partial^2 w}{\partial t^2}=q(x,t) \quad (8)$$

$$EI\left[A_0\frac{\partial^3 w}{\partial x^3}-B_0\frac{\partial^2\phi}{\partial x^2}\right]-\rho I\left[A_0\frac{\partial^3 w}{\partial x\partial t^2}-B_0\frac{\partial^2\phi}{\partial t^2}\right]+GAC_0\phi=0 \quad (9)$$

WhereA^0, B^0 andC^0 are the sti ness coe cients in governing equations. The associated consistent natural boundary conditions obtained are of following form along the edges x = 0 and x = L.

$$EI\left[\frac{\partial^3 w}{\partial x^3}-A_0\frac{\partial^2\phi}{\partial x^2}\right]-\rho I\left[\frac{\partial^3 w}{\partial x\partial t^2}-A_0\frac{\partial^2\phi}{\partial t^2}\right]=0 \quad (10)$$

Where w is Prescribed.

$$EI\left[\frac{\partial^2 w}{\partial x^2}-A_0\frac{\partial\phi}{\partial x}\right]=0 \quad (11)$$

Where $\frac{\partial w}{\partial x}$ is Prescribed.

$$EI\left[A_0\frac{\partial^2 w}{\partial x^2}-B_0\frac{\partial\phi}{\partial x}\right]=0 \quad (12)$$

Where ϕ is Prescribed.

The þexural behaviour of beam is given by solution of above equations 8 and 9 by discarding all terms containing time derivatives and satisfying the associate boundary conditions. The stiffness coefficient used in governing equations Equations 8,9,10,11 and 12 are described as below:

$$A_0=\left[12\cosh\left(\frac{1}{2}\right)-24\sinh\left(\frac{1}{2}\right)-\frac{1}{5}\cosh\left(\frac{1}{2}\right)\right] \quad (13)$$

$$B_0=6\left[\sinh(1)-1\right]-200\cosh^2\left(\frac{1}{2}\right)+432\sinh\left(\frac{1}{2}\right)\cosh\left(\frac{1}{2}\right)+\frac{1}{21}\cosh^2\left(\frac{1}{2}\right) \quad (14)$$

$$C_0=\frac{1}{2}\left[\sinh(1)+1\right]+16\cosh^2\left(\frac{1}{2}\right)-36\sinh\left(\frac{1}{2}\right)\cosh\left(\frac{1}{2}\right)+\frac{1}{5}\cosh^2\left(\frac{1}{2}\right) \quad (15)$$

2.5 The general solution of governing equilibrium equations of the Beam

The general solution for transverse displacement w(x) andű(x) can be obtained from equation 8 and 9 by discarding the terms containing time (t) derivatives. Integrating and rearranging the equation 8, we obtained the following equation

$$\frac{d^3 w}{dx^3}=A_0\frac{d^2\phi}{dx^2}+\frac{Q(x)}{D} \quad (16)$$

Where,Q(x) is generalised shear force for beam.

$$Q(x)=\int_0^x q dx + k1 \quad (17)$$

The second governing equation 9 can be written as:

$$\frac{d^3 w}{dx^3}=\frac{B_0}{A_0}\frac{d^2\phi}{dx^2}-\beta\phi \quad (18)$$

Now using equations 16 and 18 a single equation in terms ofű is obtained as:

$$\frac{d^2\phi}{dx^2}-\lambda^2\phi=\frac{Q(x)}{\alpha D} \quad (19)$$

$$w(x)=\left(\frac{x^5}{L^5}-\frac{10}{3}\frac{x^3}{L^3}+\frac{7}{3}\frac{x}{L}\right)+10\frac{E}{G}\frac{h^2}{L^2}\frac{A_0^2}{C_0}\times\left(\frac{\sinh\lambda x}{\lambda^2L^2\cosh\lambda L}-\frac{x}{L}\frac{1}{\lambda^2L^2}-\frac{1}{6}\frac{x^3}{L^3}+\frac{1}{6}\frac{x}{L}\right) \quad (25)$$

The general solution of equation 19 is as follows:

$$\phi = k_2 \cosh \lambda x + k_3 \sinh \lambda x - \frac{Q(x)}{\beta D} \qquad (20)$$

Where the constants Ų, ϐ, ↄ and D used in above equations are given below:

$$\alpha = \left(\frac{B_0}{A_0} - A_0\right) \qquad \beta = \left(\frac{GAC_0}{DA_0}\right)$$
$$\lambda^2 = \frac{\beta}{\alpha} \qquad D = EI$$

The equation of transverse displacement w(x) is obtained by substituting the expression of ű(x) in equation 18 and integrating it thrice with respect to x. The general solution for w(x) is obtained as
follows:

$$EIw(x) = \int\int\int\int q dxdxdxdx + \frac{D}{\lambda^3}\left(\frac{B_0}{A_0}\lambda^2 - \beta\right)$$
$$(k_2 \sinh \lambda x + k_3 \cosh \lambda x)\frac{k_1 x^3}{6} + k_4\frac{x^2}{2} + k_5 x + k_6 \qquad (21)$$

where k1, k2, k3, k4, k5 and k6 are the constants of integration and can be obtained by applying the boundary conditions of the beams.

3. Illustartive example:
In order to prove the e ciency of the present theory, the following numerical examples are considered.
The following material properties for beam are used.
Material properties:
1. Modulus of Elasticity E=210GPa
2. Poissions ratio = 0.30
3. Density =7800 Kg/m3

3.1 Example 1: Simply supported beam with uniformly varying load q(x) =q0(x/L)
A simply supported beam with the origin of beam on left end support at x = 0. The beam is subjected to uniformly varying load of q(x) over the span L on surface z = h/2 acting in the z direction given by

$$q(x) = q_0\left(\frac{x}{L}\right)$$

The boundary conditions associated with this problem are as follows:
Simple supports:

The axial displacement, stresses and transverse shear stress obtained based on above solutions are as follows

4.Numerical results:
The numerical results for axial displacement, transverse displacement, bending stress and transverse shear stress are presented in following non dimensional form and the values are presented in Table 1 and Table 2

$$\overline{w} = \frac{10Ebh^3}{q_0 L^4} \qquad \overline{u} = \frac{Eb}{q_0 h}u$$
$$\phi(x) \quad \overline{\sigma}_x = \frac{b\sigma_x}{q_0} \qquad \overline{\tau}_{zx} = \frac{b\tau_{zx}}{q_0}$$
$$\left(\frac{\cosh \lambda x}{\lambda L \cosh \lambda L} - \frac{1}{\lambda^2 L^2} - \frac{1}{2}\frac{x^2}{L^2} + \frac{1}{6}\right) \qquad (26)$$

$$\begin{aligned}\overline{u} = &-\frac{z}{h} \times \frac{1}{10} \times \frac{L^3}{h^3} \times \left(5\frac{x^4}{L^4} - 10\frac{x^2}{L^2} + \frac{7}{3}\right)\\ &-\frac{z}{h}\frac{E}{G}\frac{L}{h}\frac{A_0^2}{C_0}\left(\frac{\cosh \lambda x}{\lambda L \cosh \lambda L} - \frac{1}{\lambda^2 L^2}\frac{1}{2}\frac{x^2}{L^2} + \frac{1}{6}\right)\\ &+\frac{A_0}{C_0}\frac{E}{G}\frac{L}{h} \times \left[\sinh\left(\frac{z}{h}\right) - \frac{4}{3}\frac{z^3}{h^3}\cosh\left(\frac{1}{2}\right)\right]\\ &\times \left(\frac{\cosh \lambda x}{\lambda L \cosh \lambda L} - \frac{1}{\lambda^2 L^2} - \frac{1}{2}\frac{x^2}{L^2} + \frac{1}{6}\right)\end{aligned} \qquad (27)$$

$$\begin{aligned}\overline{\sigma}_x = &\left[\left(\frac{20x^3}{L^3} - \frac{20x}{L}\right) + 10\frac{E}{G}\frac{h^2}{L^2}\frac{A_0^2}{C_0}\left(\frac{\sinh \lambda x}{\cosh \lambda L} - \frac{x}{L}\right)\right] \times\\ &\left(-\frac{z}{h} \times \frac{1}{10} \times \frac{L^2}{h^2}\right) + \frac{A_0}{C_0} \times \frac{E}{G}\left(\frac{\sinh \lambda x}{\cosh \lambda L} - \frac{x}{L}\right) \times\\ &\left[\sinh\left(\frac{z}{h}\right) - \frac{4}{3}\frac{z^3}{h^3}\cosh\left(\frac{1}{2}\right)\right]\end{aligned} \qquad (28)$$

$$\begin{aligned}\overline{\tau}_{zx}^{CR} = &\frac{A_0}{C_0}\frac{L}{h}\left[\cosh\left(\frac{z}{h}\right) - \frac{4z^2}{h^2}\cosh\left(\frac{1}{2}\right)\right]\\ &\left(\frac{\cosh \lambda x}{\cosh \lambda L} - \frac{1}{\lambda^2 L^2} - \frac{1}{2}\frac{x^2}{L^2} + \frac{1}{6}\right)\end{aligned} \qquad (29)$$

$$\begin{aligned}\overline{\tau}_{zx}^{EE} = &\left[\frac{60x^2}{L^2} - 20 + 10\frac{E}{G}\frac{h^2}{L^2}\frac{A_0^2}{C_0}\left(\frac{\lambda L \cosh \lambda x}{\cosh \lambda L} - 1\right)\right] \times\\ &\left[\frac{1}{80}\frac{L}{h}\left(\frac{4z^2}{h^2} - 1\right)\right] + \frac{A_0}{C_0}\frac{E}{G}\frac{h}{L}\left(\frac{\lambda L \cosh \lambda x}{\cosh \lambda L} - 1\right) \times\\ &\left[\frac{1}{48}\cosh\left(\frac{1}{2}\right)\left(\frac{16z^4}{h^4} - 1\right) - \cosh\left(\frac{z}{h}\right) + \cosh\left(\frac{1}{2}\right)\right]\end{aligned} \qquad (30)$$

International Journal of Engineering Research
Volume No.5 Issue: Special 3, pp: 565-571

ISSN:2319-6890(online),2347-5013(print)
27-28 Feb. 2016

Table1:Non-DimensionalAxialDisplacement(u)at(x=0.75L,z= h/2), TransverseDeþection(w)at(x=0.75L, z=0.0), Axial stress(ůx) at (x=0.75L, z=h/2), Maximum Transverse shear stresses (ŰCR
zx) and (ŰEEzx) at (x=0.0, z=0.0) of simply supported beam subjected to Varying load for Aspect Ratio 4

Source	Model	$\overline{w}$	$\overline{u}$	$\overline{\sigma}_x$	$\overline{\tau}_{zx}^{CR}$	$\overline{\tau}_{zx}^{EE}$
Present	HPSDT	0.6874	-5.5903	-5.4451	-0.9992	0.9959
Dahake	TSDT	0.6891	-5.6081	-5.4438	-1.0320	0.9968
Krishna Murty	HSDT	0.6874	-5.5902	-5.4450	-0.9988	0.9959
Timoshenko	FSDT	0.6877	-5.4807	-5.2500	-0.1436	1.0000
Bernoulli-Euler	ETB	0.5811	-5.4708	-5.2500	–	1.0000

Table2:
Non-DimensionalAxialDisplacement(u)at(x=0.75L,z=h/2), TransverseDeþection(w)at(x=0.75L, z=0.0), Axial stress(ůx) at (x=0.75L, z=h/2), Maximum Transverse shear stresses (ŰCRzx) and (ŰEEzx) at (x=0.0, z=0.0) of simply supported beam subjected to Varying load for Aspect Ratio 10

Source	Model	$\overline{w}$	$\overline{u}$	$\overline{\sigma}_x$	$\overline{\tau}_{zx}^{CR}$	$\overline{\tau}_{zx}^{EE}$
Present	HPSDT	0.5981	-85.7800	-33.0076	-2.5005	2.4984
Dahake	TSDT	0.5983	-85.8249	-33.0371	-2.5801	2.4987
Krishna Murty	HSDT	0.5981	-85.7798	-33.0075	-2.4995	2.4984
Timoshenko	FSDT	0.5981	-85.4818	-32.8125	-0.8974	2.5000
Bernoulli-Euler	ETB	0.5811	-85.4818	-32.8125	–	2.5000

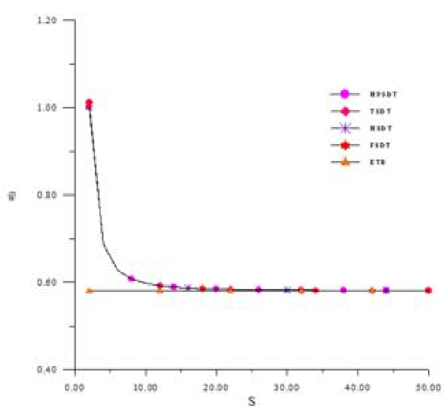

Figure 2: Variation of Transverse Displacement $\overline{w}$

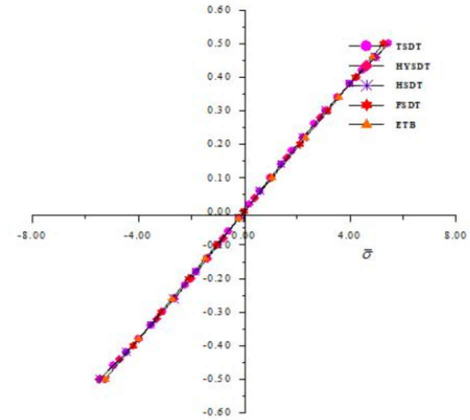

Figure 3: Variation of Maximum Axial displacement (u) for AS 04

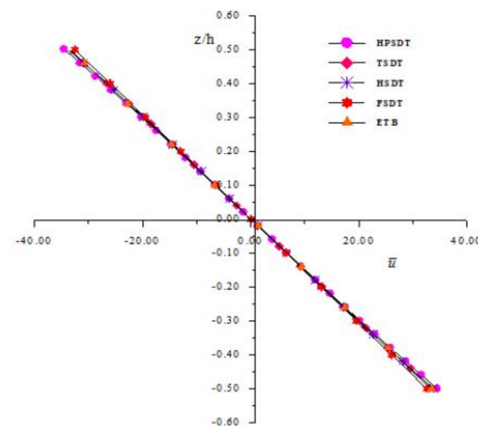

Figure 4: Variation of Maximum Axial displacement (u) for AS 10

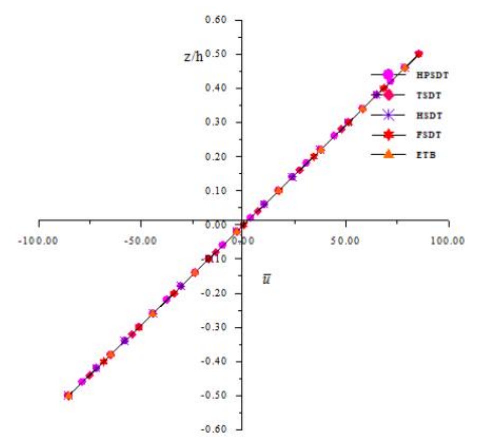

Figure 5: Variation of Maximum Axial stress (ůx) for AS 04

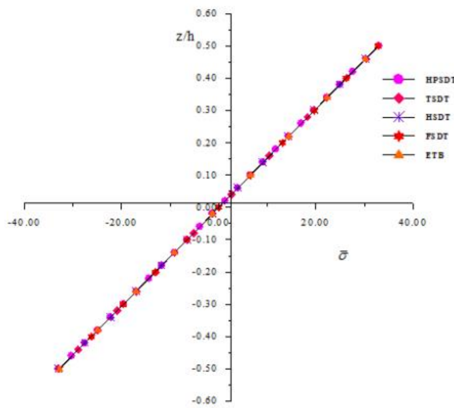

Figure 6: Variation of Maximum Axial stress (ůx) for AS 10

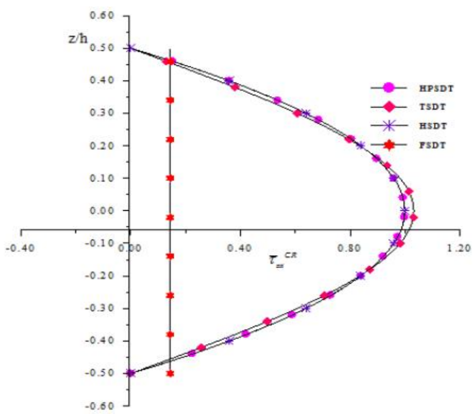

Figure 7: Variation of Transverse shear stress (ŰCR zx) for AS 04

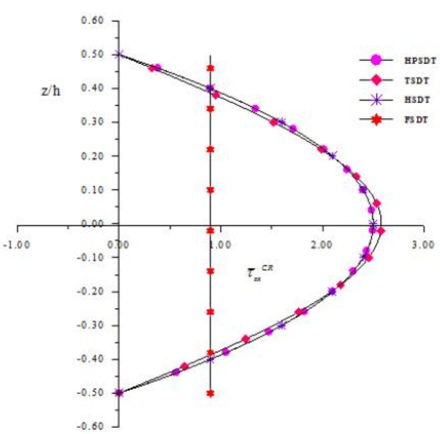

Figure 8: Variation of Transverse shear stress (ŰCR zx) for AS 10

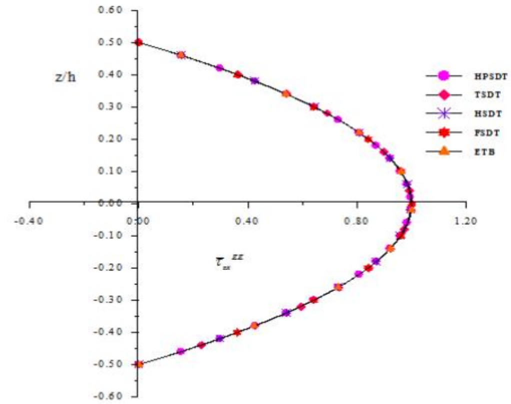

Figure 9: Variation of Transverse shear stress(ŰEEzx) for AS 04

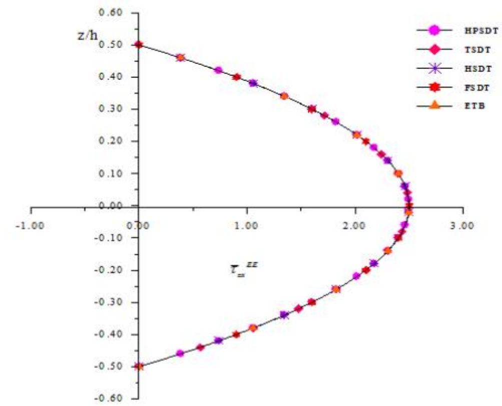

Figure10: VariationofTransverseshearstress(ŰEEzx) for AS 10

5.Concluding remark:

From the static þexural analysis of simply supports beam following conclusion are drawn:

1. The result of maximum transverse displacement obtained by present theory is in excellent agreement with those of other equivalent reýned and higher order theories. The variation of w for AS 4 and 10 are presented as shown in Figure 2.

2. From Figure 3 and Figure 4, it can be observed that, the result of axial displacement for beam subjected to varying load varies linearly through the thickness of beam for AS 4 and 10 respectively.

3. The maximum non dimensional axial stresses ůx for AS 4 and 10 varies linearly through the thickness of beam as shown in Figure 5 and Figure 6 respectively.

4. The transverse shear stress is obtained directly by constitutive relation. Figure 7, 8, 9 and Figure10 shows the through thickness variation of transverse shear stress for thick isotropic beam for AS 4 and 10. From this ýg it can be observed that, the transverse shear stress satisfy the zero condition at top (z=h/2) and at bottom (z=-h/2) surface of the beam.

References

i. *Goldenviezer, A. L., Methods for Justifying and Refining the Theory Shells, Journal of Applied Mathematics and Mechanics, Vol. 32, No.4, pp. 704-718, 1968.*

ii. *Kilchevskiy, N. A., Fundamentals of the Analytical Mechanics of Shells, NASA TT F-292, Washington, D.C., pp. 80-172, 1965.*

iii. *Donnell L. H., Beams, Plates and Shells, McGraw-Hill Book Company. New York, 1976.*

iv. *VlasovV.Z.,andLeontev,U.N., Beams, Plates and Shells on Elastic Foundations, Translated from Russian by Barouch, A., and edited by Pelz,T.Israelprogramforscientifictranslations Ltd., Jerusalem, Chapter 1, pp. 1-8, 1960.*

v. *Sayir M., and Mitropoulos, C., On Elementary Theories of Linear Elastic Beams, Plates and Shells, Zeitschrift fur Angewandte Mathematik und Physik, Vol. 31, No.1, pp. 1-55, 1980.*

vi. *Rankine W. J. M., A Manual of Applied Mechanics, R. Griffin and Company Ltd., London, U. K., pp. 342-344, 1858.*

vii. *Bresse J. A. C., Cours de Mecanique Applique, Paris: Mallet-bachelier, (1866 2nd ed.), Gauthier-Villars, Paris, 1859.*

viii. *Rebello C. A., Bert, C. W., and Gordaninejad, F., Vibration of Bi modular Sandwich Beams with Thick Facings: A New Theory and Experimental Results, Journal of Sound and Vibration, Vol. 90, No.3, pp. 381-397, 1983.*

ix. *Rayleigh, Lord, Theory of Sound, Macmillan Publishers, London, U.K, 1880.*

x. *Timoshenko, S. P., On the Correction for Shear of the Differential Equation for Transverse Vibrations of Prismatic Bars, Philosophical Magazine, Series 6, Vol.41, pp. 742-746, 1921.*

xi. *Cowper G. R., The Shear Coefficients in Timoshenko Beam Theory,ASMEJournalofApplied Mechanics, Vol. 33, pp. 335-340, 1966.*

xii. *Cowper G. R., On The Accuracy of Timoshenkos Beam Theory, ASCE journal of the Engineering Mechanics Division, Vol. 94, No. EM 6, pp. 1447-1453, 1968.*

xiii. *Murty, A. V. K., Vibration of Short Beams, AIAA Journal, Vol. 8, pp. 34-38, 1970.*

xiv. *Murty, A. V. K., Analysis of Short Beams, AIAA Journal, Vol. 8, pp.2098- 2100, 1970.*

xv. *Stephen, N. G., and Levinson, M., A Second Order Beam Theory, Journal of Sound and Vibration, Vol. 67, pp. 293-305, 1979.*

xvi. *Soler, A. I., Higher Order Effects in Thick RectangularElastic Beams, International Journal of Solids and Structures, Vol.4, No.7, pp.723-739, 1968.*

xvii. *Tsai, H. and Soler, A. I., Approximate Theory for Locally Loaded Plane Orthotropic Beams, International Journal of Solids and Structures, Vol. 6, No.8, pp. 1055-1068, 1970.*

xviii. *Levinson M., A New Rectangular Beam Theory, Journal of Sound and Vibration, Vol. 74, pp. 81-87, 1981.*

xix. *Levinson M., Further Results of a New Beam Theory, Journal of Sound and Vibration, Vol. 77, pp. 440-444, 1981.*

xx. *Levinson M., On Bickfords Consistent Higher Order Beam Theory, Mechanics Research Communications, Vol. 12, pp.1-9, 1985.*

xxi. *Levinson M., Consistent and Inconsistent Higher Order Beam and Plate Theories: some Surprising Comparisons, Proceedings of Euromech Colloquium 219 on Refined Dynamical Theories of Beams, Plates and Shells and Their Applications, Kassel, F. R. Germany, pp. 122130, 1986.*

xxii. *Krishna Murty, A. V., Toward a Consistent Beam Theory, AIAA Journal, Vol. 22, pp. 811816, 1984.*

xxiii. *Ajay G. Dahake, Dr. Yuwaraj M. Ghugal Flexure of Thick Simply Supported Beam Using Trigonometric Shear Deformation Theory, International Journal of Scientific and Research Publications, Volume 2, Issue 11, November 2012.*

xxiv. *Ajay G. Dahake, Dr. Yuwaraj M. Ghugal Flexure of thick beams using refined shear deformation theory, International Journal of civil and structural engineering, Volume 3, No. 2, 2012.*

xxv. *Yuwaraj M. Ghugal, Ajay G. Dahake Flexure of Simply Supported Thick Beams Using Refined Shear Deformation Theory, World Academy of Science, Engineering and Technology International Journal of Civil, Structural, Construction and Architectural Engineering Vol:7, No:1, 2013.*

xxvi. *Ghugal Y. M. and Sharma, R., A Hyperbolic Shear Deformation Theory for Flexure and Vibration of Thick Isotropic Beams, International Journal of Computational Methods, Vol. 6, No. 4, pp. 585-604, 2009.*

xxvii. *Ghugal, Y. M. and Sharma, R., A Refined Shear Deformation Theory for Flexure of Thick Beams, Latin American Journal of Solids and Structures, Vol. 8, pp. 183-193, 2011.*

xxviii. *Sayyad A. S. and Ghugal Y. M., Flexure of Thick Beams using New Hyperbolic Shear Deformation Theory, International Journal of Mechanics, Vol. 5, No. 3, pp. 113-122, 2011.*

xxix. *Nimbalkar V. N. and Dahake A. G Displacement and stresses for Thick Beam using New Hyperbolic Shear Deformation Theory, International Journal of pure and applied research in Engineering and technology, Vol. 3, No. 9, pp. 120130, 2015.*

xxx. *Tupe D.H. and Dahake A.G Trigonometric Shear Deformation Theory For Thick Simply Supported Beams, International Journal of Research in Engineering and Technology, Vol. 4, No. 13, pp. 476-483, 2015.*