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Dynamic Analysis of Planar Revolute type Multilink Robots with Flexible Joints

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Abstract:Generally Kinematic and Dynamic analysis of robots are performed without modeling the flexibility of kinematic Joints, because of which the results obtained are not accurate. For better analysis and control the correct estimation and inclusion of joint effects is imminent. This paper attempts to determine the effect of joint flexibility on two link manipulator when the end effector moves in a predetermined path (straight line). The joint is considered flexible and the link is assumed to be rigid. The mathematical model so developed is found to be highly non-linear, therefore numerical solutions based on Taylor series approximation were developed. Flexibility in the joint is assumed as a combination of dampers and springs.The two links of Revolute- Revolute Type of Planar robot with revolute joints with specified dimensions is built in the ADAMS software and is made to follow a specified end effector motion i.e. (straight line) for a given time period. The joint displacements so found are used as the inputs to calculate the respective torques developed using MATLAB software. The obtained results are tabulated and compared to provide information on the effectiveness of flexibility.

Keywords: Joint Flexibility, Inverse Dynamics, Taylor Series Approximation.

I. INTRODUCTION

In robotics, the Kinematic and Dynamic analysis are made with assumption that the joints are rigid, but in reality flexibility occurs due to the following reasons i.e. when the actuator shaft is provided a torque, the shaft transmits the rotation to the link at the joint. As total weight of the links act at the joint, resulting in torsional stresses in the actuator shaft. This results in stiffness between the rotations of the link and the actuator due to these torsional stresses. Ideally the link pin (journal) is expected to rotate within the bearing oil. But in practice, the link pin (journal) touches the surface of the bearing inner walls (hub). This causes wear and tear and significantly affects the nature of the dynamic responses developed. The effect of wear and tear has to be introduced in the mathematical analysis for better understanding of physical phenomenon and accuracy of the torques developed. One method of solving this problem is to assume the joint has flexibility and this flexibility can be shown as a combination of equivalent stiffness and damping.

II.LITERATURE REVIEW

Martin Corless et al [1] have analyzed flexibility of link and actuator shaft as a system of two bodies having stiffness and damping in between their angular displacement. The two rotations i.e., rotation of the body and rotation of the actuator shaft are two different functions related to each other. Kakizaki et al[2] have used Newton-Euler's method to model robotic manipulator for dynamic analysis. Iraj-Hassanzadeh et al [3] have used numerical methods to approximate the angular velocity and angular acceleration by assuming the function quadratic over small finite interval of time Δt. Farid et al [4] have adopted Bernoulli-Euler beams to model flexible links of the manipulator, which provides a means of calculating the load-carrying and deflection characteristics of link. Yan-Ru Hu et al [5] mentioned the requirement of forward dynamic solution in motion and force control of robot arms in the presence of joint flexibility. Jankowski et al [6] mentioned the requirement of inverse dynamic solution for the manipulator end effector to follow a preplanned path specified in the robot task space. Farid et al [4] mentioned the importance of the inverse dynamic solution, as the input is needed at the actuator to perform the task of obtaining the desired output, which is the main objective of the robot.

III.PROBLEM DEFINITION

The objective of this paper is to obtain a mathematical model for two-link RR type planar manipulator with and without flexible joints and find the effect of flexibility of the joints on the motor torques developed.

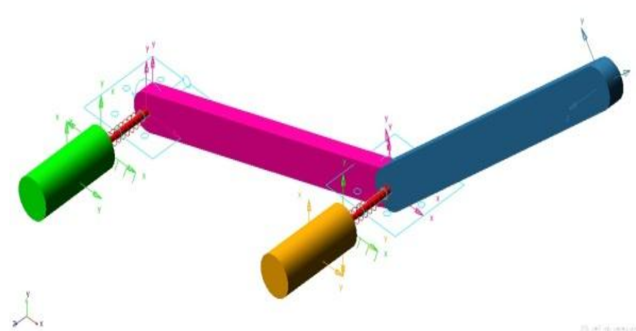

Figure 1: Two link rigid manipulator

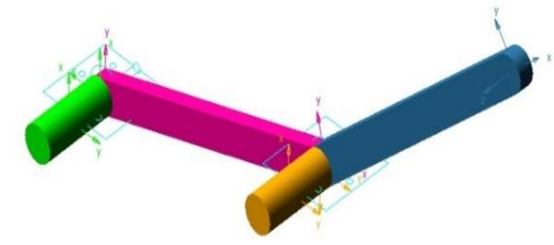

Figure 2: Two link flexible manipulator

The link parameters assumed are as follows

Table:1

Dimensions of the Manipulator	
length of Link l_1	206.15 mm
length of Link l_2	206.15 mm
Moment of Inertia of I_{G_1}	$1527\ \mathrm{kg - mm^2}$
Moment of Inertia of I_{G_2}	$1527\ \mathrm{kg - mm^2}$
Mass of Link 2 is m_2	0.36 kg
Mass of Link 2 is m_2	0.36 kg
Stiffness of link 1 is K_1	20 Nm/rad
Stiffness of link 2 is K_2	20 Nm/rad
Damping coefficient of link 1 is B_1	300 Nm-s/rad
Damping coefficient of link 2 is B_2	300 Nm-s/rad
The initial value of joint 1 angle at initial time	$4.9\ rad$
The initial value of joint 2 angle at initial time	$2.6\ rad$
Breadth of the links (b)	0.05m
Thickness of the links (t)	0.025m
Density of the material used (ρ)	7801 kg/m^3
Diameter of shaft at the joints (d)	0.05m

The forward and inverse dynamic solutions are to be found for manipulators with rigid joints,this has to be done very carefully as it includes time variant parameters such as moment of inertia which changes from time to time apart from
the input and output variable functions and finally the flexibility is introduced as a combination of springs and dampers. If the models were linear then Laplace transforms could be used to find solutions, when initial conditions were zero. But for non-linear and time variant systems Laplace transforms cannot be used. Hence taylor series approximation is used to find both the forward and inverse dynamic solutions. Once the solution is found, a graphical output is to be generated with given graphical input so that the performance of this derived solution could be estimated. Error of not more 6° in the rotation of the output can be tolerated

IV.SOLUTION METHODOLOGY

The purpose is to find the required torque function to manipulate the robot for a given trajectory. As a case study two link rigid manipulator is analyzed and then extended to flexible joints. Newton-Euler's approach is used to model the dynamic system.

4.1 Mathematical model for a Two linkRevolute-Revolute (RR) type Planar Manipulator with rigid joints

(Torque due to inertia of the link) + (Torque due to the inertia of the actuator shaft) + (Torque due to the weight of the link) = (Torque given at the actuator) (refer figure 1)

$$I\ddot{\theta}_t + J\ddot{\theta}_t + WC_\theta = \tau_t \qquad 4.1$$

The dynamic equation is given in equation 4.1, the nature ofa single link manipulator, when it is extended to two link rigid manipulator the resultant equation is given by equation 4.2, where $m_1, m_2,$ are the respective masses, θ_1, θ_2 are respective angular displacements of two links, τ_1 , τ_2 are respective torques generated at the two links.$I_{1,2}$, $I_{2,2}$ are the moment of inertia of the links at respective joints. J_1, J_2 are the polar moment of inertias of the actuator shaft at the respective joints

$$\begin{bmatrix} I_{1,2} & 0 \\ 0 & I_{2,2} \end{bmatrix} \begin{Bmatrix} \ddot{\theta_1} \\ \ddot{\theta_2} \end{Bmatrix} + \begin{bmatrix} J_1 & 0 \\ 0 & J_1 \end{bmatrix} \begin{Bmatrix} \ddot{\theta_1} \\ \ddot{\theta_2} \end{Bmatrix} + \begin{bmatrix} m_1 & m_2 \\ 0 & m_2 \end{bmatrix} g \begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} \tau_1 \\ \tau_2 \end{Bmatrix} \qquad 4.2$$

Where $C_1=\frac{l_1}{2}\cos\theta_1$ and $C_2=l_2\cos\theta_2$ 4.3

$$I_{1,2} = I_{G1} + m_1\left(\frac{l_1}{2}\cos\theta_1\right)^2 + I_{G2} + m_2\left(l_1^2 + \frac{l_2^2}{4} + l_1 l_2 \cos\theta_2\right) \qquad 4.4$$

$$I_{2,2} = I_{G2} + m_2\left(\frac{l_2}{2}\cos\theta_2\right)^2 \qquad 4.5$$

Inverse solution:Let the angular displacement be θ_t at time t. For small change in the time Δt, left hand and right hand surface angular displacements are given by using Taylor's series.

$$\{\theta\}_{t+\Delta t} = \{\theta\}_t + \frac{\Delta t}{1}\{\dot{\theta}\}_t + \frac{(\Delta t)^2}{2!}\{\ddot{\theta}\}_t + \frac{(\Delta t)^3}{3!}\{\dddot{\theta}\}_t + \cdots \ 4.6$$

$$\{\theta\}_{t-\Delta t} = \{\theta\}_t - \frac{\Delta t}{1}\{\dot{\theta}\}_t + \frac{(\Delta t)^2}{2!}\{\ddot{\theta}\}_t - \frac{(\Delta t)^3}{3!}\{\dddot{\theta}\}_t + \cdots 4.7$$

By taking the change in function quadratic in time interval Δt, third and higher ordered terms are neglected. The expressions for $\{\dot{\theta}\}_t$ and $\{\ddot{\theta}\}_t$ are given by the above equations as follows.

$$\{\dot{\theta}\}_t = \frac{\{\theta\}_{t+\Delta t} - \{\theta\}_{t-\Delta t}}{2\,\Delta t} 4.8$$

$$\{\ddot{\theta}\}_t = \frac{\{\theta\}_{t-\Delta t} - 2\{\theta\}_t + \{\theta\}_{t+\Delta t}}{(\Delta t)^2} \qquad 4.9$$

When the above values of equation 4.9 are substituted in equation 4.1 results in

Forward solution:

$$\{\theta\}_{t+\Delta t} = (\Delta t)^2\{\tau\}_t - ([I]_\theta + [J])^{-1}(\Delta t)^2[W]\{C\}_\theta + 2\{\theta\}_t - \{\theta\}_{t-\Delta t} \qquad 4.10$$

$$\{\{\tau\}_t = \left(\frac{[I]_\theta + [J]}{(\Delta t)^2}\right)\{\theta\}_{t-\Delta t} - 2\left(\frac{[I]_\theta + [J]}{(\Delta t)^2}\right)\{\theta\}_t + [W]\{C\}_\theta +$$
$I\theta$+$[J]\Delta t2\theta t$+Δt 4.11

and when it is assumed that

$$\{\theta\}_{-\Delta t} = \{\theta\}_0, \{\theta\}_{(n+1)\Delta t} = \{\theta\}_{n\Delta t} \qquad 4.12$$

Hence the torque generated can be calculated provided $\{\theta\}_{(n+1)\Delta t}$ is known

4.2 Mathematical model for a Two link Revolute -Revolute (RR) type Planar Manipulator with two flexible revolute joints

Newton-Euler's approach is used to model the dynamic system. (Torque due to the inertia of the link) + (Relative torque at the link due to the damping between the link and the actuator shaft) + (Relative torque at the link due to the stiffness between the link and the actuator) + (Torque due to the weight of the link) = 0

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$$I(\ddot{\theta}_t) + B(\dot{\theta}_t - \dot{\emptyset}_t) + K(\theta_t - \emptyset_t) + WC_\theta = 0 \qquad 4.13$$
$$I\ddot{\theta}_t + B\dot{\theta}_t + K\theta_t + WC_\theta = B\dot{\emptyset}_t + K\emptyset_t$$

When it is extended to a two link manipulator, it results in

$$\begin{bmatrix} I_{1,2} & 0 \\ 0 & I_{2,2} \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} +$$
$$\begin{bmatrix} m_1 & m_2 \\ 0 & m_2 \end{bmatrix} g \begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{Bmatrix} \dot{\emptyset}_1 \\ \dot{\emptyset}_2 \end{Bmatrix} + \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \begin{Bmatrix} \emptyset_1 \\ \emptyset_2 \end{Bmatrix} \; 4.14$$

(Torque due to the inertia of the actuator shaft) + (Relative torque at the actuator shaft due to the damping between the link and the actuator shaft) + (Relative torque at the actuator shaft due to the stiffness between the link and the actuator) =
(Torque given at the actuator)

$$J\ddot{\emptyset}_t + B(\dot{\emptyset}_t - \dot{\theta}_t) + K(\emptyset_t - \theta_t) = \tau_t \qquad 4.15$$
$$J\ddot{\emptyset}_t + B\dot{\emptyset}_t + K\emptyset_t = B\dot{\theta}_t + K\theta_t + \tau_t$$

$$\begin{bmatrix} J_1 & 0 \\ 0 & J_1 \end{bmatrix} \begin{Bmatrix} \ddot{\emptyset}_1 \\ \ddot{\emptyset}_2 \end{Bmatrix} + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{Bmatrix} \dot{\emptyset}_1 \\ \dot{\emptyset}_2 \end{Bmatrix} + \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \begin{Bmatrix} \emptyset_1 \\ \emptyset_2 \end{Bmatrix} =$$
$$\begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} + \begin{Bmatrix} \tau_1 \\ \tau_2 \end{Bmatrix} \qquad 4.16$$

Where B_1, B_2, K_1, K_2 are equivalent damping and stiffness between respective links and actuators. In the same fashion as the expressions for $\{\ddot{\emptyset}\}_t$ and $\{\dot{\emptyset}\}_t$ and $\{\ddot{\theta}\}_t$ and $\{\dot{\theta}\}_t$, are approximated using taylor series and substituted in the governing equations 4.13 and 4.15 results in the following Forward and Inverse solution.

Forward solution:

$$\{\theta\}_{t+\Delta t} = \left[\left(\frac{[B]}{2\,\Delta t}\right) - \left(\frac{[J]}{(\Delta t)^2} + \frac{[B]}{2\,\Delta t}\right) 2\,\Delta t [B]^{-1} \left(\frac{[I]}{(\Delta t)^2} + \right.\right.$$

B2 Δt−12J/Δt2∅t−Δt−2J/Δt2+J/Δt22 Δt[B]−1K∅t+B2 Δt+J/Δt2+B2 Δt2 Δt[B]−1I/Δt2−B2 Δtθt−Δt+J/Δt2+B2 Δt2 Δt[B]−1K−2I/Δt2−Kθt+J/Δt2+B2 ΔtWCθ−τt
4.16

Inverse solution:

$$\{\tau\}_t =$$
$$\left(2\frac{J}{(\Delta t)^2}\right)\{\emptyset\}_{t-\Delta t} - \left[\left(2\frac{J}{(\Delta t)^2}\right) + \left(\frac{J}{(\Delta t)^2}\right) 2\,\Delta t [B]^{-1}[K]\right]\{\emptyset\}_t +$$
$$\left[\left(\frac{[B]}{2\,\Delta t}\right) + \left(\frac{[J]}{(\Delta t)^2} + \frac{[B]}{2\,\Delta t}\right) 2\,\Delta t[B]^{-1}\left(\frac{[I]}{(\Delta t)^2} - \frac{[B]}{2\,\Delta t}\right)\right]\{\theta\}_{t-\Delta t} +$$
$$\left[\left(\frac{[J]}{(\Delta t)^2} + \frac{[B]}{2\,\Delta t}\right) 2\,\Delta t[B]^{-1}\left([K] - 2\frac{[I]}{(\Delta t)^2}\right) - [K]\right]\{\theta\}_t +$$
$$\left(\frac{[J]}{(\Delta t)^2} + \frac{[B]}{2\,\Delta t}\right)[W]\{C\}_\theta + \left[\left(\frac{[J]}{(\Delta t)^2} + \frac{[B]}{2\,\Delta t}\right) 2\,\Delta t[B]^{-1}\left(\frac{[I]}{(\Delta t)^2} + \right.\right.$$

B2Δt−B2Δtθt+Δt 4.17

and assuming $\{\theta\}_{-\Delta t} = \{\emptyset\}_{-\Delta t} = \{\emptyset\}_0 = \{\theta\}_0$ and $\{\theta\}_{(n+1)\Delta t} = \{\theta\}_{n\Delta t}, \{\emptyset\}_{(n+1)\Delta t} = \{\emptyset\}_{n\Delta t}$ 4.18

hence resultant generated torque can be calculated if the resultant angular displacement is known.

V. RESULTS & DISCUSSIONS

When the endeffector is made to move in a specified path i.e straight line, the torque required to move the two links is estimated based on taylor series approximation for both the cases i.e. when the joints are assumed to be Rigid and Flexible. The end effector is made to move on a straight line path defined by straight line $x = 100 + 5\,t$ over a period of time. The results obtained for this case are tabulated below. When the end effector moves on a straight line and the joints are assumed to be rigid, the variation of joint displacement over a period of time is given in Figure 3 and Figure 4
hows the variation of torque generated at both the joints, i.e when the joints are assumed to be rigid.It is observed that the torque at the first joint is higher than that at the second joint. The possible reason is that the first joint balances both the links while the second joint balance only second link.

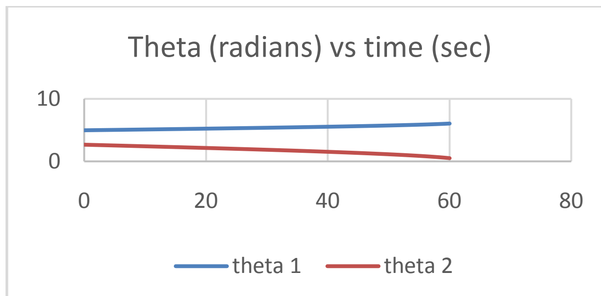

Figure 3: Angular displacement (radians) variations of joints with time

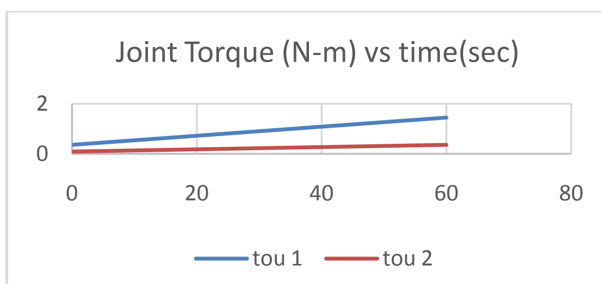

Figure 4: (rigid) Joint Torque variation with time

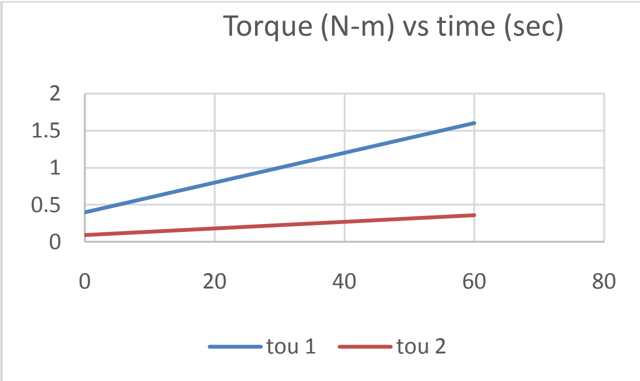

Figure 5: Variation of Joint Torque (when only stiffness is considered)

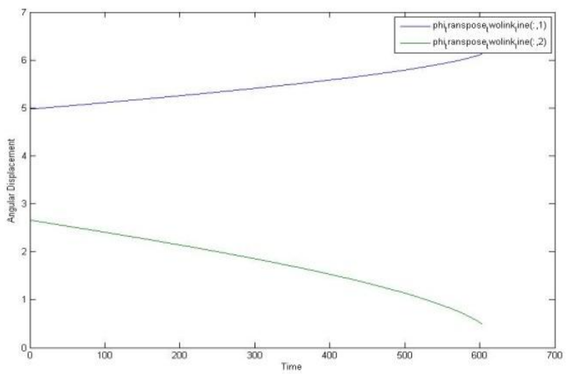

Figure 6: Variation of angular displacement (φ) in radians of the actuating shaft when joint is considered flexible (only stiffness)

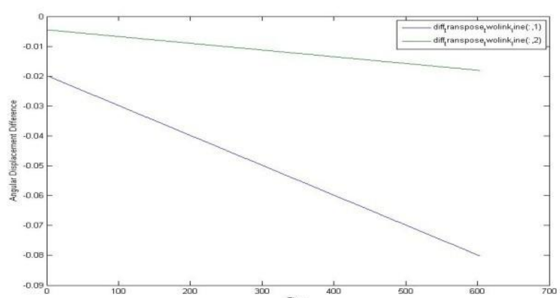

Figure 7: Difference of angular displacements (in radians) of Joint and actuating shaft ($\theta - \varphi$) when joint is flexible(only stiffness)

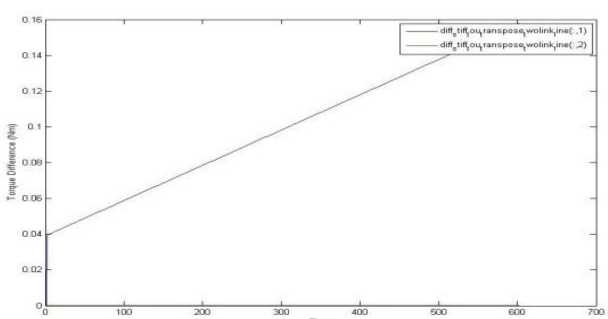

Figure 8: Difference in τ(N-m) due to flexibility (only stiffness)

When the rigid joints are replaced with flexible joints (where flexibility is only due to stiffness of the joint)

When the rigid joints are replaced with flexible (only Stiffness $K_{1,2} = 20$ Nm/rad) joints and end effector is made to move on the same path ,the Figure 5 shows the variation of torques generated at the two joints when joint is flexible and Figure 6specifies the variation of the angular displacement of the actuating shaft. i.e. there is a difference in the rotations of the motor actuating shaft(φ) and the joint connected to it(θ). When the end effector moves in a straight line, the angular displacement of the joint is given by θ, where as the joint is actuated by an individual motor (φ) , it is connected to the motorshaft. In the ideal case when the joint is considered rigid the angular displacement of the shaft connected to the motor and the joint actuated by the motor will be same. But if the joint is considered as flexible then there will be difference in the angular displacements of the respective motor shaft and the joint actuated by it. This difference over a period of time is given by Figure 6.This difference in the angular displacements in between shaft and joint causes variation in the torques generated from that of the torques generated by considering the joints to be rigid. This variation in torque is given by Figure 7. In this case it was assumed that the flexibility is because of only stiffness at the respective joints.

When the rigid joints are replaced with flexible joints (where flexibility is because of stiffness and damping of the joint)

and the end effector is made to move along the same straight line and joint flexibility is considered as an equivalent combination of Dampers and springs. Figures 8 to 11 gives the variation of generated torques, difference in angular displacements of respective shafts and the joints actuated by them and the variation of torque to that of torque generated by considering the joint to be rigid. The joint flexibility is assumed due to Stiffness ($K_{1,2} = 20$ Nm/rad) and Damping coefficient ($B_{1,2} = 300$ Nm-s/rad)

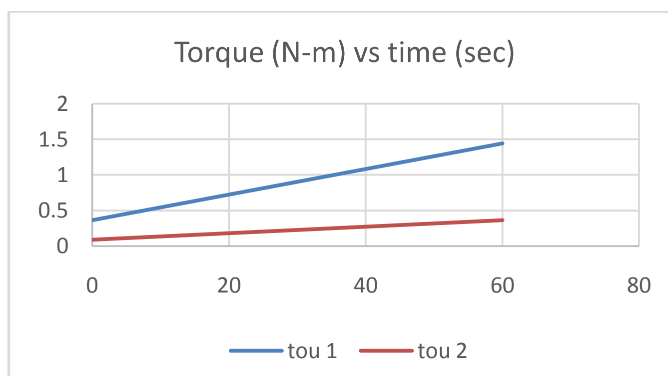

Figure 9: Variation of Torque at the two flexible joints (N-m)

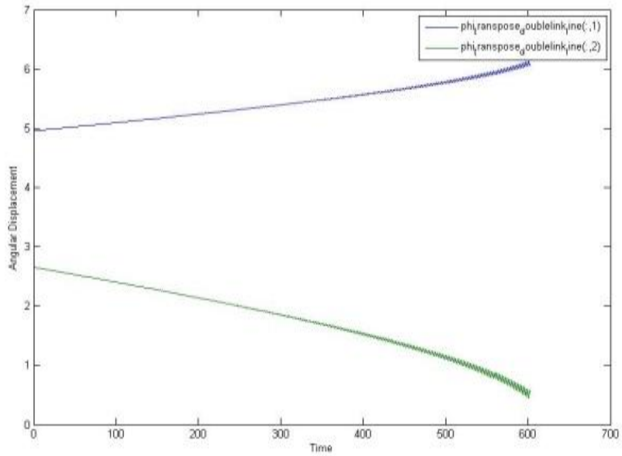

Figure 10: Variation of angular displacement (φ) in radians of the actuating shaft due to flexibility (stiffness+damping)

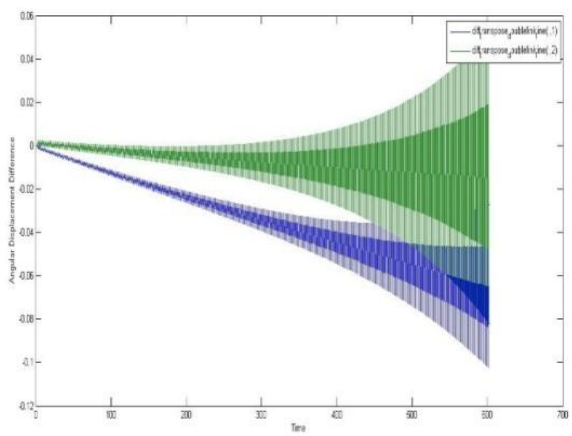

Figure 11: Difference of angular displacement (in radians) of Joint and actuating shaft ($\theta - \varphi$)due to flexibility (K+B)

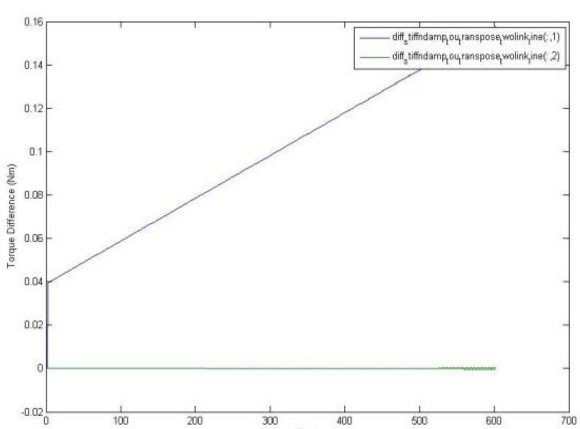

Figure 12: Difference in τ (N-m) due to flexibility (K+B) at the joints

The results obtained are tabulated below from tables 2 to 6

Table 2:
Variation of Angular displacements and Torques generated for Link 1(consideringstiffness) with Straight Line Motion with respect to the rigid joint

Link 1	In Radians			N-m		
	Theta	Phi	Difference	Torque	Torque (f)	Difference
Max Value	6.038	6.118	-0.020	1.443	1.601	1.58E-01
Std Dev	0.291	0.308	0.017	0.313	0.348	3.43E-02

A percentage change of 10.9 in torque and 0.93 in angular displacement is observed

Table 3:
Variation of Angular displacements and Torques generated for Link 2 (consideringstiffness) with Straight Line Motion with respect to the rigid joint

Two Link	Theta	Phi	Difference	Torque	Torque(f)	Difference
	(radians)			(N-m)		
Max Value	2.651	2.656	-0.004	0.360	0.360	8.58E-06
Std Dev	0.582	0.578	0.003	0.078	0.078	1.15E-06

A percentage change of 0.0003 in torque and 0.64 in angular displacement is observed.

It is observed that the average torque difference of perfect joint and flexible joint is very large at link 1 when compared with that of link 2.

Table 4:
Variation of Angular displacements and Torques generated for Link 1(considering both stiffness and damping) with Straight Line Motion with respect to the rigid joint

Link 1	Theta	Phi	Difference	Torque	Torque (f)	Difference
	(radians)			(N-m)		
Max Value	6.038	6.135	0	1.443	1.601	1.58E-01
Std Dev	0.291	0.309	0.022	0.313	0.348	3.43E-02

A change of 10.93 % in torque and 0.63 % in angular displacement is observed

Table 5:
Variation of Angular displacements and Torques generated for Link 2(considering both stiffness and damping) with Straight Line Motion with respect to the rigid joint

Link 2	Theta	Phi	Difference	Torque	Torque(f)	Difference

	(radians)			(N-m)		
Max Value	2.651	2.651	0.052	0.360	0.360	5.33E-04
Std Dev	0.582	0.578	0.024	0.078	0.078	1.86E-04

A change of 0.0006 % in torque and 0.423 % in angular displacement is observed

Table 6:
Effect of flexibility on generated Joint Torques (N-m)

Two Links		Average τ	Average Difference(τ)		% change in τ with respect to Rigid joint	
			K	K+B	K	K+B
	Link 1	0.9	0.098	0.099	10.935	10.935
	Link 2	0.23	5.72E-07	1.47E-06	0.0002	0.0006

VI. CONCLUSIONS

Based on the results obtained this paper concludes that the torque requirement increased by 10% when flexibility is considered as an equivalent combination of Stiffness and damping. Therefore it is imperative to take in to the effect of joint flexibility for kinematic analysis of multi link manipulators for better judgment of applied Torque.

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