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A Fuzzy Model with Changing Deterioration Rate

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***Abstract:* A fuzzy inventory model is developed for deteriorating items having shortages at transportation time. Two different rates of deterioration are allowed at the time of transportation and demand. Graded mean representation method is used for defuzzification.**
Keywords: Inventory , deterioration , shortages, replenishment, Triangular fuzzy number, graded mean representation method

1. *Introduction*

The deterioration of goods is a realistic phenomenon in many inventory systems. The controlling and regulating of deteriorating items is a measure problem in any inventory system. Certain products deteriorate during their normal storage period. Hence while developing an optimal inventory policy for such products, the loss of inventory due to deterioration cannot be ignored. The researchers have continuously modified the deteriorating inventory models so as to become more practicable and realistic. The analysis of deteriorating inventory model is initiated by Ghare and Schrader[2] with a constant rate of decay. Several researchers have extended their idea to different situations in deterioration on inventory model. The models for these type products have been developed by Jhuma Bhowmick, G.P.Samanta[3], Liqun Ji[5], U.K.Misra, S.K.Sahu, Bhaskar Bhaula and L.K.Raju[6].

Most of inventory models taking deterioration at the node of supply channel have been developed by researchers. But deteriorating loss in the stage of transportation becomes one of the important contract parameter during the negotiation process. Zhao Xiao Yu, zheng Yi, JIA Tao [7] developed two-phase deteriorating inventory model with the assumption that the on-hand inventory level held by the retailer is demanded at a constant rate.

In conventional inventory models, various types of uncertainties are classically modeled using the approaches from the probability theory. However, there are uncertainties that cannot be appropriately treated by usual probabilistic models. To define inventory optimization tasks in such environment and to interpret optimal solutions, fuzzy set theory rather than probability theory is more convenient. Considering the fuzzy set theory in inventory modeling renders an authenticity to the model formulated since fuzziness is the closest possible approach to reality.

Jaggi [8] discussed the fuzzy inventory model for deteriorating items with time-varying demand and shortages. They used Graded mean representation , Signed distance and Centroid methods to defuzzify the total cost function. Syed [9] developed a fuzzy inventory model without shortages using Signed distance method. They used fuzzy triangular number for both ordering cost and holding cost. Umap [10] formed a fuzzy EOQ model for deteriorating items with two warehouses. He considered fuzzy numbers for the parameters such as holding cost and deteriorating cost for two warehouses. He used signed distance method and function principle method for defuzzification of total inventory costs as well as optimum order quantity.

An inventory model by allowing the shortages at the time of transportation with different deterioration rates occur at transportation and storage stages is considered. This model is solved analytically to determine the optimal cycle time and numerical example is provided to illustrate this model. Some of the parameters are considered as triangular fuzzy numbers for fuzzy case. For defuzzification of the total cost function and optimum order quantity, Graded mean representation method is used.

2. *Preliminaries*

2.1Basic Definitions

2.1.1 Fuzzy Set

If X is a collection of objects denoted generically by x, then a fuzzy set $\tilde{A}$ in X is defined as a set of ordered pairs $\tilde{A} = \{(x; \mu_{\tilde{A}}(x)) / x \epsilon X\}$, where $\mu_{\tilde{A}}(x)$ is called the membership function for the fuzzy set $\tilde{A}$. The membership function maps each element of X to a membership grade between 0 and 1(included)

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2.1.2 α - Cut

The set of elements that belong to the fuzzy set $\widetilde{A}$ at least to the degree of α is called the α level set or α – cut. (i.e),

$\widetilde{A}^{(\alpha)} = \{x \in X: \mu_{\widetilde{A}}(x) \geq \alpha\}$

2.2 Fuzzy Number

Fuzzy numbers are of great importance in fuzzy systems.

2.2.1 Fuzzy Number

A fuzzy subset $\widetilde{A}$ of the real line R with membership function $\mu_{\widetilde{A}} : R \to [0,1]$ is called a fuzzy number if

i. $\widetilde{A}$ is normal, (i.e), there exist an element x_0 such that $\mu_{\widetilde{A}}(x_0) = 1$

ii. $\widetilde{A}$ is fuzzy convex,

(i.e), $\mu_{\widetilde{A}}[\lambda x_1 + (1-\lambda)x_2] \geq \mu_{\widetilde{A}}(x_1) \wedge \mu_{\widetilde{A}}(x_2)$
$x_1, x_2 \in R, \forall \lambda \in [0,1]$

iii. $\mu_{\widetilde{A}}$ is upper continuous.

iv. supp $\widetilde{A}$ is bounded, where supp $\widetilde{A} = \{x \in R: \mu_{\widetilde{A}}(x) > 0\}$

2.2.2 Generalized Fuzzy Number

Any fuzzy subset of the real line R, whose membership function $\mu_{\widetilde{A}}$ satisfies the following conditions, is a generalized fuzzy number

i. $\mu_{\widetilde{A}}(x)$ is a continuous mapping from R to the closed interval $[0,1]$

ii. $\mu_{\widetilde{A}}(x) = 0, \quad -\infty < x \leq a_1$

iii. $\mu_{\widetilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$

iv. $\mu_{\widetilde{A}}(x) = 1, \quad a_2 \leq x \leq a_3$

v. $\mu_{\widetilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$

vi. $\mu_{\widetilde{A}}(x) = 0, \quad a_4 \leq x < \infty$, where a_1, a_2, a_3, a_4 are real numbers

2.2.3 Triangular Fuzzy Number

The fuzzy set $\widetilde{A} = (a_1, a_2, a_3)$, where $a_1 \leq a_2 \leq a_3$ and defined on R, is called triangular fuzzy number, if the membership function of $\widetilde{A}$ is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

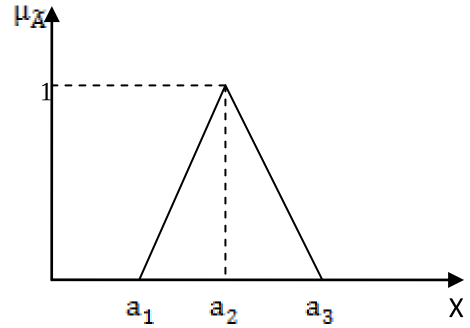

Fig 1 : Triangular Fuzzy Number (a_1, a_2, a_3)

2.2.4 Operations of Triangular Fuzzy Number

Consider two triangular fuzzy numbers $\widetilde{A} = (a_1, a_2, a_3)$, $\widetilde{B} = (b_1, b_2, b_3)$

i. The addition of $\widetilde{A}$ and $\widetilde{B}$ is

$\widetilde{A} + \widetilde{B} = (a_1, a_2, a_3) + (b_1, b_2, b_3)$
$= (a_1 + b_1, a_2 + b_2, a_3 + b_3)$,

where $a_1, a_2, a_3, b_1, b_2, b_3$ are real numbers.

ii. The multiplication of $\widetilde{A}$ and $\widetilde{B}$ is

$\widetilde{A} \times \widetilde{B} = (c_1, c_2, c_3)$, where
$T = \{a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3\}$,
$c_1 = \min T, c_2 = a_2 b_2, c_3 = \max T$

If $a_1, a_2, a_3, b_1, b_2, b_3$ are all non zero positive real numbers, then

$\widetilde{A} \times \widetilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3)$

iii. $[-\widetilde{B}] = -(b_1, b_2, b_3) = (-b_3, -b_2, -b_1)$ then the subtraction of $\widetilde{B}$ from $\widetilde{A}$ is

$\widetilde{A} - \widetilde{B} = (a_1, a_2, a_3) - (b_1, b_2, b_3)$
$= (a_1 - b_3, a_2 - b_2, a_3 - b_1)$,

where $a_1, a_2, a_3, b_1, b_2, b_3$ are real numbers.

iv. $\frac{1}{\widetilde{B}} = \widetilde{B}^{-1} = \left(\frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}\right)$, where b_1, b_2, b_3 are all non zero positive real numbers, then division of $\widetilde{A}$ and $\widetilde{B}$ is $\frac{\widetilde{A}}{\widetilde{B}} = \frac{(a_1, a_2, a_3)}{(b_1, b_2, b_3)} = \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right)$

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v. For any real number

$$k, \quad k\widetilde{A} = \begin{cases} (ka_1, ka_2, ka_3), \text{if } k > 0 \\ (ka_3, ka_2, ka_1), \text{if } k < 0 \end{cases}$$

2.3 Defuzzification

Defuzzification is the conversion of a fuzzy quantity to a crisp quantity. Defuzzification methods obtain the representative value of a fuzzy set.

2.3.1 Graded Mean Representation Method

Let $\widetilde{A}$ be a fuzzy number with left reference function L and right reference function R. Let L^{-1} and R^{-1} be the inverse functions of L and R respectively.

The graded mean integration representation of $(\widetilde{A})$ is defined by

$$p(\widetilde{A}) = \frac{\frac{1}{2}\int_0^1 h[L^{-1}(h) + R^{-1}(h)]dh}{\int_0^1 h\, dh} \text{ with } 0 < h \le 1$$

By the above formula, the graded mean representations of triangular fuzzy number $\widetilde{A} = (a_1, a_2, a_3)$ is given by

$$p(\widetilde{A}) = \frac{a_1 + 4a_2 + a_3}{6}$$

3. Notations

T	-	The length of the cycle
t_d	-	The length of the transportation time
$Q(t)$	-	The quantity level at time t
Q	-	The order quantity per cycle
$I(t)$	-	The inventory level at time t
Q_I	-	The inventory level at time t_d
Q_S	-	The back order level at time t_d
θ_1	-	The deterioration rate at the time of transportation
θ_2	-	The deterioration rate at $[t_d, T]$
α	-	The constant demand rate
h	-	The holding cost per unit time
p	-	The purchase cost per unit time
f	-	The transportation cost per unit time
u	-	The order processing cost
d	-	The deterioration cost per unit time
s	-	The shortage cost per unit time
FC	-	The transportation cost per cycle
DC	-	The deterioration cost per cycle
SC	-	The shortage cost per cycle
HC_T	-	The holding cost in transit per cycle
HC_{NT}	-	The holding cost in non transit per cycle
PC	-	The purchase cost per cycle
TC	-	The total inventory cost per cycle
$\widetilde{TC}_{dG}(T)$	-	The defuzzified value of $\widetilde{TC}(T)$ by Graded mean representation method
$\widetilde{Q}_{dG}$	-	The deffuzzified value of $\widetilde{Q}$ by Graded mean representation method

4. Assumptions

1. There is a fixed transport time t_d before the goods were shipped from vendor to retailer.
2. The shortages are allowed at the time of transportation.
3. There is no replacement or repair of deteriorated items during the cycle under consideration.
4. The demand rate is deterministic and constant.
5. The inventory system operates for a prescribed time horizon
6. The lead time is zero.

5. Model Formulation

The problem of inventory system with change in deterioration rate is considered. This system goes as follows. At time t = 0, the order is placed for the order quantity Q. There is a fixed transportation time t_d before the goods were shipped from vendor to buyer. During t_d, the deterioration rate of goods is θ_1. The shortages are allowed during this t_d. The quantity received at t_d by buyer after transportation is used to make up the shortages that accumulated in $[0, t_d]$ and demand in $[t_d, T]$. There is again deterioration in $[t_d, T]$ with a different rate θ_2 where the inventory level dropped to zero at T. The entire process is repeated.

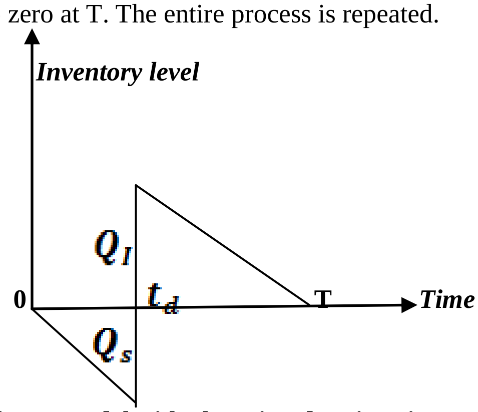

Fig 2 : Model with changing deterioration rate

5. CRISP MODEL

The change of quantity level can be described by the following equations

$$\frac{dQ(t)}{dt} = \begin{cases} -\theta_1 Q(t), & 0 < t \le t_d \\ -\theta_2 Q(t) - \alpha, & t_d \le t \le T \end{cases} \quad (1)$$

with the boundary conditions $Q(t_d) = Q_I + Q_S$, $Q(T) = 0$

Consider

$$\frac{dQ(t)}{dt} = -\theta_1 Q(t), \qquad 0 < t \le t_d$$

$$\frac{dQ(t)}{Q(t)} = -\theta_1 dt$$

$$Q(t) = e^{-\theta_1 t + c_1}$$

Then

$$\mathbf{Q(t) = e^{-\theta_1(t-t_d)} Q(t_d), \qquad 0 < t \le t_d}$$

Consider

$$\frac{dQ(t)}{dt} = -\theta_2 Q(t) - \alpha, \qquad t_d \le t \le T$$

$$Q(t)e^{\theta_2 t} = -\alpha \frac{e^{\theta_2 t}}{\theta_2} + C_2$$

$$Q(t) = \frac{-\alpha}{\theta_2} + \frac{\alpha}{\theta_2} e^{\theta_2 T} e^{-\theta_2 t}$$

$$\mathbf{Q(t) = \frac{\alpha}{\theta_2}\left(e^{-\theta_2(t-T)} - 1\right), \qquad t_d \le t \le T}$$

The solutions of (1) are given by

$$Q(t) = \begin{cases} e^{-\theta_1(t-t_d)} Q(t_d), & 0 < t \le t_d \\ \frac{\alpha}{\theta_2}\left(e^{-\theta_2(t-T)} - 1\right), & t_d \le t \le T \end{cases} \qquad (2)$$

By considering the continuity at t_d ,

$$Q_I + \alpha t_d = \frac{\alpha}{\theta_2}\left(e^{-\theta_2(t_d-T)} - 1\right) = Q(t_d) \qquad (3)$$

$$I(t_d) = Q_I = \frac{\alpha}{\theta_2}\left(e^{-\theta_2(t_d-T)} - 1\right) - \alpha t_d$$

$$Q = Q(t_d) + \theta_1 t_d = Q_I + Q_s + \theta_1 t_d$$

Therefore the order quantity

$$Q = \frac{\alpha}{\theta_2}\left[e^{-\theta_2(t_d-T)} - 1\right] + \theta_1 t_d \qquad (4)$$

Then (2) becomes

$$Q(t) = \begin{cases} \frac{\alpha}{\theta_2} e^{-\theta_1(t-t_d)}\left(e^{-\theta_2(t_d-T)} - 1\right), & 0 < t \le t_d \\ \frac{\alpha}{\theta_2}\left(e^{-\theta_2(t-T)} - 1\right), & t_d \le t \le T \end{cases} \qquad (5)$$

The change of inventory level can be described by the following equations

$$I(t) = \begin{cases} e^{-\theta_1(t-t_d)} I(t_d), & 0 < t \le t_d \\ \frac{\alpha}{\theta_2}\left(e^{-\theta_2(t-T)} - 1\right), & t_d \le t \le T \end{cases}$$

$$I(t) = \begin{cases} \frac{\alpha}{\theta_2} e^{-\theta_1(t-t_d)}\left(e^{-\theta_2(t_d-T)} - 1\right) - \alpha t_d e^{-\theta_1(t-t_d)}, & 0 < t \le t_d \\ \frac{\alpha}{\theta_2}\left(e^{-\theta_2(t-T)} - 1\right), & t_d \le t \le T \end{cases} \qquad (6)$$

The transportation cost per cycle $FC = f\, t_d$ (7)

The order processing cost $OC = u$ (8)

The deterioration cost per cycle $DC = d(Q - \alpha T)$

$$= d\left(\frac{\alpha}{\theta_2}\left[e^{-\theta_2(t_d-T)} - 1\right] + \theta_1 t_d - \alpha T\right) \qquad (9)$$

The holding cost in transit per cycle

$$HC_T = h\int_0^{t_d} I(t)dt$$

$$= \frac{h\alpha e^{\theta_1 t_d}\left[e^{-\theta_1 t_d} - 1\right]}{\theta_1}\left(t_d - \frac{\left[e^{-\theta_2(t_d-T)} - 1\right]}{\theta_2}\right)$$

$$= \frac{h\alpha\left(1 - e^{\theta_1 t_d}\right)\left[\theta_2 t_d - e^{-\theta_2(t_d-T)} + 1\right]}{\theta_1\theta_2} \qquad (10)$$

The holding cost in non-transit per cycle

$$HC_{NT} = h\int_{t_d}^{T} I(t)dt$$

$$= \frac{h\alpha}{\theta_2}\int_{t_d}^{T}\left[e^{-\theta_2(t-T)} - 1\right]dt$$

$$= \frac{h\alpha}{\theta_2}\left[\frac{-1}{\theta_2} - T + \frac{e^{-\theta_2(t_d-T)}}{\theta_2} + t_d\right]$$

$$= \frac{h\alpha}{\theta_2^2}\left(e^{-\theta_2(t_d-T)} - 1 + \theta_2 t_d - \theta_2 T\right) \qquad (11)$$

The shortage cost per cycle $SC = s\,\alpha\, t_d$ (12)

The purchasing cost per cycle $PC = pQ$

$$= p\left(\frac{\alpha}{\theta_2}\left[e^{-\theta_2(t_d-T)} - 1\right] + \theta_1 t_d\right) \qquad (13)$$

The total inventory cost

$$TC(T) = ft_d + u + d\left(\frac{\alpha}{\theta_2}\left[e^{-\theta_2(t_d-T)} - 1\right] + \theta_1 t_d - \alpha T\right) + \frac{h\alpha\left(1 - e^{\theta_1 t_d}\right)\left[\theta_2 t_d - e^{-\theta_2(t_d-T)} + 1\right]}{\theta_1\theta_2} + \frac{h\alpha}{\theta_2^2}\left(e^{-\theta_2(t_d-T)} - 1 + \theta_2 t_d - \theta_2 T\right) + s\alpha t_d + p\left(\frac{\alpha}{\theta_2}\left[e^{-\theta_2(t_d-T)} - 1\right] + \theta_1 t_d\right) \qquad (14)$$

$$\frac{d[TC(T)]}{dT} = \frac{d\alpha}{\theta_2^{\,2}}\left[e^{-\theta_2(t_d-T)} - \theta_2^{\,2}\right] - \frac{h\alpha\left(1 - e^{\theta_1 t_d}\right)}{\theta_1\theta_2^{\,2}}\left(e^{-\theta_2(t_d-T)}\right) + \frac{h\alpha\left[e^{-\theta_2(t_d-T)} - \theta_2^{\,2}\right]}{\theta_2^3} + \frac{p\alpha}{\theta_2^{\,2}}\left(e^{-\theta_2(t_d-T)}\right) \qquad (15)$$

$$\frac{d^2[TC(T)]}{dT^2}=\frac{d\alpha}{\theta_2^{\,3}}\left(e^{-\theta_2(t_d-T)}\right)-\frac{h\alpha\left(1-e^{\theta_1 t_d}\right)}{\theta_1\theta_2^{\,3}}\left(e^{-\theta_2(t_d-T)}\right)+\frac{h\alpha}{\theta_2^{\,4}}\left(e^{-\theta_2(t_d-T)}\right)+\frac{p\alpha}{\theta_2^{\,3}}\left(e^{-\theta_2(t_d-T)}\right) \quad (16)$$

For a minimum of

$$TC(T), \frac{d[TC(T)]}{dT}=0 \text{ and } \frac{d^2[TC(T)]}{dT^2}>0 \; at\; T=T^*$$

6. *FUZZY MODEL*

Here some of the parameters are fuzzy numbers namely $\tilde{\alpha}$, $\tilde{f}$, $\tilde{u}$, $\tilde{d}$, $\tilde{h}$, $\tilde{s}$, $\tilde{c}$.

Let

$$\tilde{\alpha}=(\alpha_1,\alpha_2,\alpha_3), \tilde{f}=(f_1,f_2,f_3), \quad \tilde{u}=(u_1,u_2,u_3),$$
$$\tilde{d}=(d_1,d_2,d_3), \tilde{h}=(h_1,h_2,h_3), \tilde{s}=(s_1,s_2,s_3), \tilde{p}=(p_1,p_2,p_3)$$

The total cost of the system per unit time in fuzzy sense is given by

$$\widetilde{TC}(T)=\tilde{f}t_d+\tilde{u}+\tilde{d}\left(\frac{\tilde{\alpha}}{\theta_2}\left[e^{-\theta_2(t_d-T)}-1\right]+\theta_1 t_d-\tilde{\alpha}T\right)+\frac{\tilde{h}\tilde{\alpha}\left(1-e^{\theta_1 t_d}\right)\left[\theta_2 t_d-e^{-\theta_2(t_d-T)}+1\right]}{\theta_1\theta_2}$$
$$+\frac{\tilde{h}\tilde{\alpha}}{\theta_2^2}\left(e^{-\theta_2(t_d-T)}-1+\theta_2 t_d-\theta_2 T\right)+\tilde{s}\tilde{\alpha}t_d+\tilde{p}\left(\frac{\tilde{\alpha}}{\theta_2}\left[e^{-\theta_2(t_d-T)}-1\right]+\theta_1 t_d\right) \quad (17)$$

Then $\widetilde{TC}(T)=\left(TC_1(T),TC_2(T),TC_3(T)\right)$ (18)

The fuzzy total cost is defuzzified by graded mean representation method.

$$TC_{dG}(T)=\frac{1}{6}\left[TC_1(T)+4TC_2(T)+TC_3(T)\right]$$

For the total cost function $TC_{dG}(T)$ to be minimum,

$$\frac{\partial TC_{dG}(T)}{\partial T}=0 \text{and} \frac{\partial^2 TC_{dG}(T)}{\partial T^2}>0$$

7. Numerical Example

Crisp Model Let

$\alpha=500$, $t_d=0.3$ year, $\theta_1=0.5$/year, $\theta_2=1.5$/year,

h = Rs.30 / unit/year, d = Rs.52 /unit/year,

s=Rs.50/unit/year, p = Rs.25 /unit/year,

f = Rs800/month, u = Rs500/cycle

The solution of crisp model

$TC(T)=Rs.7405.50$,

$T=0.578year$,

Quantity Q = 173 units/year

Fuzzy ModelLet

$\tilde{a}=(450,500,550)$, $\tilde{f}=(750,800,850)$, $\tilde{u}=(475,500,525)$,

$\tilde{d}=(50,52,54)$, $\tilde{h}=(28,30,32)$, $\tilde{s}=(48,50,52)$,

$\tilde{p}=(20,25,30)$

By Graded Mean Representation Method,

$TC_{dG}(T)=Rs.7378.90$

$T=0.577year$,

Quantity Q = 258 units/year

8. *Conclusion*

An appropriate model for deteriorating things with different rates of deterioration is developed. When making the inventory decision for deteriorating items, it is a reasonable to take the transit deterioration loss into account. For fuzzy model the demand, transportation cost, ordering cost, holding cost, deterioration cost, purchasing cost, shortage cost are represented by triangular fuzzy numbers. Graded mean representation method is used for defuzzification. This proposed inventory model can be extended to deal with time varying demand, quality discount, inflation and others.

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