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Brink Depth at Free Overfall – A Review

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Abstract :***The sudden drop of river bed level is known as Free over fall and acts as a control section. Thus it can be used for the estimation of discharge flowing through the river/stream/open channels. The depth of flow just at the free over fall is known as Brink depth or End depth, by measuring which the discharge can be estimated by correlating it with the critical depth of flow. In fact the discharge can easily be estimated if the critical depth of flow is known and there is a correlation between the brink depth and the critical depth of flow.In the present paper the works carried out in this field have been studied till date. It is also mentioned what work can be carried out in future by studying and analyzing the past work.***

Keywords: Open channel flow, Free over fall, Brink depth, Critical depth

1. INTRODUCTION

The study of free over fall is important because it can be used as a flow measuring device in open channel flow. A free over fall is characterized by a channel followed by a sudden drop in its bed as shown in Fig 1. It also shows a uniform flow followed by gradually varied flow and the rapidly varied flow, at the end of which brink depth occurs just at the free over fall.

Flow over a free over fall separates in the form of nappe at the sharp edge or drop and leads to rapidly varying flow with an appreciable curvature of streamlines, which causes a non hydrostatic pressure distribution. Due to this reason the depth of flow at the brink is smaller than critical depth of flow and termed as 'END DEPTH' or 'BRINK DEPTH'. Such type of drop occurs at city drainage system and drop type hydraulic structures used in irrigation engineering like notches, falls etc. At the brink or end of a channel, the pressure at the upper and lower ends (A and B in fig 1) of the falling nappe is atmospheric and varying nearly parabolic.

Applying the momentum equation between the critical section and section at free over fall, we have the following general equation for prismatic channels.

$$\Upsilon A_C\overline{Z}_C - \Upsilon A_b\overline{Z}_b - F_f = \rho Q\,(V_b - V_c)$$

Such over fall acts as a control section having unique relationship between brink depth or end depth and discharge and so used as a flow measuring device for different shapes of channel. The ratio of the end-depth to the critical depth known as End Depth Ratio (EDR) offers a possibility to predict the discharge and study of erosion near the brink of a free overfall.

Thus the computation of end depth and its analysis has always been acquired much practical importance.

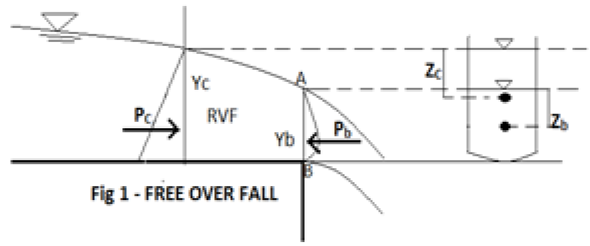

Fig 1 - FREE OVER FALL

Several researches, experimental and analytical works, have carried out on different shapes of channels for both subcritical and supercritical flow condition. Analytical attempts have been made by many investigators in the past for computation of end depth, among them most of the approaches are based on application of momentum equation with some assumptions and few are based on energy consideration and water surface profile at the end section. The numerical solution of two dimensional flows for an ideal fluid has also been attempted adopting various finite element techniques. To obtain three dimensional flow characteristics VOF (Volume of Fluid) model has also been applied. The effect of bed roughness and slope on the flow over a free over fall has been investigated by applying various experimental and turbulence models.

2. REVIEW OF LITERATURE

This part of the paper deals with a review of past research works in this field of brink depth by various investigators for better understanding of all its aspects. Research works highlighting main assumptions, principle, equations used and modeling techniques involved along with results and conclusion have been discussed in brief.

This part is presented under following categories for different shapes of the channel.

2.1 RECTANGULAR CHANNEL

Hunter Rouse (1936) [34] was probably the first investigator to recognize the interesting features of end depth at free over fall. He carried out his experiments in mild sloping rectangular channels for subcritical flow condition. He concluded the end depth is 0.715 times the corresponding critical depth for parallel flow if the pressure on the lower and upper nappe is atmospheric.

Delleur et al. (1956) [6] studied the variation of end depth ratio (Y_e/Yc) using the data of adverse, mild and steep channels. They found that for rectangular channels the ratio (Y_e/Yc) depends only upon relative slope (S_o/S_c) for both smooth and rough surfaces. They also reported variation of pressure coefficient as a function of relative slope as follows.

K_1= 0.6 for $S_o/S_c < -5.0$

$K_1 = 0.3 + \{(1- S_o/S_c)\}/8$ for $-5.0< S_o/S_c <1$
$K_1=0.30$ for $1< S_o/S_c$ (1)

Replogle (1962) [37] carried out his investigation for rectangular channel based on several assumptions used in Diskin's momentum equation (1961). He developed similar momentum equations and showed that the effect of energy correction factor (α), momentum correction factor (β) and residual pressure was small. For rectangular free over fall, he found ratio of velocity (V_c/V_e) and depth of flow (Y_c/Y_e) 0.716 and 1.396 respectively. Replogle concluded that only the end pressure effectively contributes for the reduction of depth ratio from 1.5 for rectangular over fall to 1.428 as computed by him. The remaining variation from measured value of EDR as 0.715 may be due to inaccurately determination of α values and deviation of actual pressure distribution from parabolic assumption.

Rajaratnam and Muralidhar (1964) [38] conducted experiments on rectangular channels having smooth surface for wide range of slopes. From the experiment on horizontal channel it was found that Y_e/Yc and K_1 vary with shape of channel and for sloping channel it is a function of relative slope So/S_c.

Anderson (1967) [49] proposed a method for end depth computation with a different approach in which equation of water surface profile in the channel and in gravity fall region are separately derived and matched at the end section. For rectangular channel the equation obtained was

$$4(Y_e/Yc)^3–6(E/Y_c)(Y_e/Y_c)^2+3=0 \qquad (2.1)$$

where E is the specific energy at the end section.
For subcritical flow the above equation reduced to,

$$4(Y_e/Yc)^3-9(Y_e/Y_c)^2+3=0 . \qquad (2.2)$$

Solution of equation (2.2) gives $Y_e/Yc = 0.694$, which is 3% less as compared to Rouse's value 0.715.

Strelkoff and Moyeri (1970) [47] investigated the free over fall at rectangular channel according to potential theory. In this method, boundary value was formulated as an integral equation and then solved numerically. Results agreed with the result of Hunter Rouse. The EDR for Fr = 1 was 0.672, thus slightly less than the corresponding experimental value.

Ferro (1992) [14] used free overfall as a discharge-measuring structure to establish the relationship between end depth and critical depth. He carried out the experiment on free over fall in a rectangular channel having different channel width values. The measurements showed that, for practical application, the relationship between Ye and pressure coefficient (K) is independent of channel width. By using this relationship, 89.9% of the estimated discharges are within ±5% of the experimental work.

Rai (1993) [42] studied the end depth problem in rectangular channel and for unconfined nappe; he obtained the average depth ratio (Y_e/Yc) as 0.712. For sloping channel the end depth ratio was found to be a function of relative slope (S_o/S_c). For relative slope of +5.0, the end depth ratio was found to be 76.14% of end depth ratio of horizontal rectangular channel. The apparent critical section was found to lie at a distance of $3.27Y_c$ upstream of end section for horizontal channel and was a function of slope S_o.Tiwari (1994) [52] developed an expression of free over fall at rectangular channel by applying momentum approach and developed computer software. Effect of weight of control volume on sloping floor was included in the derivation. For rectangular channel (bed width B), the equation developed by him was

$$K_1(Y_e/Y_C)^3–3(Ye/Y_C)+2=0 \qquad (3)$$

For horizontal channel and zero end pressure, end depth ratio was found to be 0.66667 and it was same as given by Diskin. For sloping channels, EDR was found to be a function of relative slope. Results obtained from theoretical software analysis agreed well with the experimental data obtained by him.

Khan and Steffler (1996) [23] gave a model for horizontal rectangular free over fall using vertically averaged and momentum equations. A linear longitudinal velocity distribution and quadratic vertical velocity and pressure distributions, was used for modeling flow in the vicinity of horizontal rectangular free overfall with smooth and rough beds and sharp-crested weirs with sloping upstream faces. These equations were modeled using a hybrid Petrov-Galerkin and Bubnov-Galerkin finite-element scheme. For rectangular free overfalls, the predicted water surface profiles upstream and the free jet trajectory agreed well with the measured data. The computed vertical velocity and pressure distributions at the brink and upstream of the overfall were found to be in good agreement with the measured data. While the computed longitudinal velocity distributions compared well with a two-dimensional potential flow model. The computed results for sharp-crested weirs with sloping upstream faces agree well with the measured data for an upstream weir slope of up to 270 with the horizontal. For an upstream slope of 450 and steeper and for a large weir height the predicted water surface upstream of a weir shows numerical instability.

Mittal and Desmukh (1998) [26] developed computer software in FORTAN-77 language for flow computation at rectangular free over fall. They developed a very useful calibration curve for rectangular free over fall. The curves were drawn between discharge and brink depth for different values of K (0, 0.3, 0.6 and 0.9). With the help of these curves, the discharge in an over fall can be determined immediately if the brink depth is known. The major equation for the curve derived for rectangular over fall was

$$Y_R=0.670+0.047K+0.160K^2 \qquad (4.1)$$
$$q^*=1.831-0.314K+0.320K^2 \qquad (4.2)$$

Where Q^* ($^3 q^*$) is the non dimensional discharge. Discharge per unit width was expressed as

$$q=1000[gY_b^3]^{0.3}q^* \qquad (4.3)$$

The error involved by using these equations was within 1% except for some higher values of K.

Davis et al. (1998) [5] carried out an experimental study of the free overfall from a rectangular channel for various slopes and bed roughness. The experiments were conducted in a metal rectangular flume with glass sides, 305 mm in width and 3.7 m in length. It had a painted steel bed with an n of 0.0099. The relationship between the upstream critical depth and brink depth was found to be affected by both slope and channel-bed roughness, with roughness having a greater effect at steeper slopes. Two empirical equations were proposed for calculating this relationship, the first requiring only the data of channel slope and the second requiring both channel slope and roughness data.

$$Y_e/Y_c=134.84S_0^2-12.66S_0+0.778 \quad (5.1)$$

$$Y_e/Y_c=0.846- 0.219(\sqrt{S_0}/n) \quad (5.2)$$

He found that, the first relationship was accurate in predicting 76.7% of the discharges to within 10% and was more useful for estimation of discharge if bed roughness is not known. The second relationship predicted 90% of the discharges to within 10%.

Dey (2000) [10] prepared a theoretical model to compute the end depth of a free over fall in steeply sloping rough rectangular channels based on momentum approach. Curvature of streamlines at the free surface was used to develop the differential equation for the flow profile upstream of the free over fall of a wide rectangular channel. An auto recursive method was developed to solve the equations simultaneously. Estimation of discharge from end depth and Nikuradse equivalent sand roughness was also done. Results were well accordingly with experimental observations except for some higher roughness.

Ahmed (2003) [3] developed a quasi theoretical method for determining end depth ratio and end depth discharge relationship in terms of pressure coefficient in sub critical and super critical flows for rectangular channel. The major equations developed are

$$EDR=Y_E/Y_1=3F_1/[2(1-C_P)+F_1^2]^{3/2}-[F_1^2-2C_P]^{3/2} \quad (6.1)$$

$$EDD = Q = (F_1\ Bg^{1/2}Y_E^{3/2})\ [\{[(2(1-C_P)\ +F_1^2)^{3/2}] - [(F_1^2-2C_P)^{3/2}]\}\ /\ 3F_1]^{3/2} \quad (6.2)$$

The brink pressure coefficient K was determined from experimental data. Predicted values of EDR and EDD were compared with experimental data. For subcritical flows the value of EDR was 0.78 for a confined nappe and 0.758 for an unconfined nappe. For supercritical flows EDR decreases with increase in relative slope (S_O/S_C) and Y_C/B. The predicated EDR in supercritical flow well agreed with experimental data.

Guo (2005) [17] developed a numerical iterative method for computing free rectangular over fall in a physical plane based on analytical function boundary value theory and substitution variables. The method was applied to calculate water surface profile, pressure distribution and EDR for both smooth and rough channel with a wide range of slope and upstream Froude number. The calculated values were well agreed with experimental data. The main advantage of this method was it was very fast and flexible to be applied on curved bed also.

Beirami et al. (2006) [4] prepared a theoretical model based on the free vortex theorem and the momentum equation was applied at the brink of free over falls in channels of different cross sections with sub-critical flow. The model was used to calculate the pressure head distribution, the pressure coefficient, the end depth ratio (EDR), and flow discharge at the brink. Using available experimental and theoretical results of other investigators the proposed method was examined. In rectangular channels, the proposed method gave the values of 0.7016 and 0.3033 for EDR and K, respectively. According to the values of EDR reported by other investigators had a slight difference 1% to 2% with the proposed method. The differences between the other formulas with the proposed method are about 1.5% to 3%.

Guo et al. (2008) [18] carried out both experimental and turbulent numerical modeling for the study of free over fall in rectangular channel with strip roughness. The channel was 0.4m wide, 0.4m depth and 8.4m long with plywood bottom and sides. An array of strip roughness was placed on the channel bottom had a square cross section of 6×10^{-3}m high and wide and transversely fixed. A wide range of model parameters like bed roughness, channel slope and upstream Froude number was investigated. The upstream water surface profile, velocity in the cavity between two strips and end depth were simulated, measured and discussed for various input conditions. The result showed that for a given dimension of bed roughness relative spacing of roughness affected flow condition, which decreased with increase in λ/d value in this study. For small spacing, eddies reduced the downstream discharge while such geometric effect was negligible while λ/d value was increased up to a certain range.

Tigrek et al. (2008) [53] carried out an experimental study to get a relationship between brink depth and discharge at a rectangular free over fall. A series of experiments were conducted in a tilting rectangular flume of 1m width and 12.06m length for both sub critical and super critical flow conditions. The equation derived for end depth ratio and discharge were as follows.

$$Y_e / Y_c = 0.683 \quad \text{for } F_r <= 1 \text{ with rms } =0.0341 \quad (7.1)$$

$$Y_e / Y_c = 0.773 - 0.018(\sqrt{S_0}/n) \quad \text{for } F_r > 1 \text{ with rms } =0.0708 \quad (7.2)$$

$$q=C_dY_e^{3/2} \quad (7.3)$$

where $C_d = 5.55$ for $F_r <= 1$

$$C_d = [1 / [(0.361-(0.00841(\sqrt{S_0}/n))]]^{3/2}$$

Validity of this explicit discharge brink depth equation was checked and the result agreed well with predicted values.

Mohammed et al**.** (2011) [29] carried out an experimental study to determine the effect of gravel roughness and channel slope on rectangular free over fall. The experiments were conducted in a metal rectangular flume with glass sides, 300mm in width and 10m in length. The flume was set to slopes of 0, 1/200, 1/100 respectively. For various types of roughness the end depth ratio was expressed as

$$Ye/Yc=C_1+C_2[(k/Y_c)S_0]^{0.5} \quad (8)$$

C_1 and C_2 were different for different gravel roughness distribution on bed and slope. It was found that the ratio of Ye / Yc for full bed of dimension 20cm x 30 cm having roughness of two rows of 6 mm gravel had more effect at steeper slopes. Six relationships were obtained to predict Ye / Yc. All these relationships were compared with Davis et al. (1998) and Tigrek et al.(2008) to ensure their utility and validity.

Amirabdollahian et al. (2012) [2] simulated flow over a free over fall in rectangular channel based on potential flow theory and Schwarz- Christoffel transformation. The flow had been assumed to be inviscied and ir-rotational. Considering the straight line segments simulation of the free over fall flow course a complex function was achieved which transfer the vertical and horizontal lines into a complex plane of stream function and velocity potential respectively. Based on determining the detailed flow pattern the free surface nappe formula had been derived. Results showed that use of conformal mapping was well agreed with results.

Mohammed (2013) [30] studied the effect of slopes and bed rough distribution on the free over fall in rectangular channel. The rectangular flume was 0.3m wide 0.45m deep and 10m long with glass sides. Bed roughness was made of wood 1cm diameter and 1cm height allocated in three different cases: two, three and zigzag rows. Three empirical discharge equations for free overfall depending on brink depth and slope were obtained. Three rows bed roughness having greater effect on these relationships at steeper slopes. The average values of Yc/Ye at smooth bed was greater by 4% with respect to that for bed rough at two rows, by 19% with respect to that for bed rough at zigzag rows and by 24% with respect to that for bed rough at three rows, so that values for three rows rough and horizontal channel was greater by 4% with respect to that for channel slope at 1/200 and by 14% with respect to that for channel slope at 1/100.

Hou et al. (2013) [20] investigated the free overfall in open channels with even and uneven bottom by using the mesh less smoothed particle hydrodynamics (SPH) method. For the even bottom case, subcritical, critical and supercritical flows were simulated. For the uneven bottom case, supercritical flows with different Froude numbers were considered. The free surface profiles were predicted and compared with theoretical and experimental data and it agreed well.

2.2 TRAPEZODIAL CHANNEL

Diskin (1961) [7] developed equations for the computation of end depth in exponential and trapezoidal channel using momentum principle. He assumed pressure at the end section to be zero. The basic equation developed by him as follows.

$$A_C\overline{Y}_C=(Q^2/g)[(1/A_e)-(1/A_c)] \quad (9.1)$$

Solving the above equation the above equation, relationship obtained by Diskin for horizontal trapezoidal channel:

$$(X_C+X_C^2)/(X_e+X_e^2)=(10X_C^2+20X_C+9)/[6(1+X_C^2)] \quad (9.2)$$

In which $X_C= mY_C / B$ and $X_e= mY_e / B$ with m and B as side slope and width respectively. He also suggested a simplified equation for end depth in non-dimensional form as

$$X_e=1/2\ [-1 + (1+4Z_C)^{1/2}],$$

$$\text{in which } Z_C= [6X_C(1+X_C)^3] / (9+20X_C+10X_C^2) \quad (9.3)$$

Rajaratnam (1962) [36] developed equations for exponential and trapezoidal channels with non-zero pressure at the end section by momentum principle. For trapezoidal channel,

$$X_e^5 + X_e^4 + X_e^3- [(\varphi_1 (X_C) +1)/ (K_1\varphi_2 (X_C))]\ X_e^2 - [(\varphi_1 (X_C) +1) / (K_1\varphi_2 (X_C))]\ X_e+ [(\varphi_3 (X_C)) / (K_1\varphi_2 (X_C))] = 0, \quad (9.4)$$

where $\varphi_1 (X_C) = (3+2 X_C) / [6(1+ X_C)]$

$$\varphi_2 (X_C) = (3+2 X_C)/ [(X_C+X_C^2)\ \varphi\ (1+X_C)\ X_C]$$

$$\varphi_3 (X_C) = (X_C+X_C^2)$$

He also proposed some correction in Diskin's method. Rajaratnam and Thiruvengadam gave a two parameter solution for solution of critical depth in trapezoidal channel as

$$Qm^{3/2}/ (b/2)^{5/2}=4\ [(4X_C^2+4X_C)^{3/2} / (1+2X_C)^{1/2}] \quad (9.5)$$

LHS of above equation is also a function of X_e since X_e has been shown to be a function of X_C. They gave a graph between functional relationship of Q and X_e.

Rajaratnam and Muralidhar (1970) [40] performed experiments on over falls in smooth trapezoidal channel and analyzed the data as obtained by Diskin. They showed the relationship of end depth ratio as follows.

$$Y_e/Y_C = f\ (S_O/S_C,\ mY_C /B) \quad (10)$$

Subramanya and Murthy (1987) [48] used Anderson's approach to solve end depth problem in trapezoidal channel based on energy consideration and continuity of water surface profile at the brink. Water surface profile was derived separately for the channel flow and gravity over fall and finally equated at the end section to obtain an expression for end depth. Its main advantage was it was free of any experimentally derived coefficient. He derived an expression for horizontal frictionless trapezoidal channel carrying a sub critical flow as

$$\epsilon\ (\Phi) - 4\eta -3\ f(\varphi,\eta) = 0 \quad (11.1)$$

$$\text{Where } \epsilon\ (\Phi) = 1 + ½\ [(1+\varphi) / (1+2\varphi)] \quad (11.2$$

$$\text{and} \quad f(\varphi,\eta)= (1+\varphi)^3/ [(1+\varphi\eta)^2\eta^2(1+2\varphi)] \quad (11.3)$$

Results obtained from these equations vary +- 2% experimental values.

Keller and Fong (1989) [22] solved equation for trapezoidal channel based on momentum approach and assuming non-zero pressure at the end section. They also conducted an experimental analysis on trapezoidal over fall. The predicted relationship of brink depth and discharge was compared with experimental data. The major equation developed was

$$10X_c^4 + 20X_c^3 + 9X_c^2 – (6/G_1)\ [(X_c^3+3X_c^4+3X_c^5+X_c^6)] - K_1G_1G_2(1+2X_c) = 0 \quad (12.1)$$

$$\text{In which } G_1 = X_e + X_e^2 \quad (12.2)$$

$$G_2 = [(3+2X_e)\ X_e] / (1+X_e) \quad (12.3)$$

The result showed that 40% of predictions were +- 2% of measured flow rates and 80% were within +- 3%. A calibration chart, applicable to any mild slope trapezoidal channel, was developed theoretically and checked against the data from the present study and data from previous studies.

Gupta et al. (1993) [16] carried out an experimental study on a smooth trapezoidal free over fall for positive, negative and zero slopes. They provided a calibration curve using dimensionless parameters $[Qm^{1.5}/(\sqrt{g}B^{2.5}]$ and $[e^{5.5(S)}mYe/B\ ]$ for the prediction of discharge (Q) with the help of known end-depth (Ye) and vice versa. The curve best fitted to the data, with a correlation coefficient (CR) equal to 0.99753. It was also observed that for horizontal channels, the theoretical value of the constant (i.e., slope) in the equation of the straight line X_e = (constant)(Xc) was greater than that obtained from the present investigation based on experimental data, where $X_e = mYe/B$ and $Xc = mY_c/B$.

Tiwari (1994) [52] developed an expression of free over fall at trapezoidal channel by applying momentum approach and developed computer software. Effect of weight of control volume on sloping floor was included in the derivation. For trapezoidal channel (bed width B and side slope 1 in m), the equation developed by him was same as that obtained by Diskin for horizontal trapezoidal channel with K = 0.For sloping channels, EDR was found to be a function of relative slope.

Litsa and Evangelos (1995) [24] investigated the flow over a fall in a trapezoidal channel by simulating that over a sharp-crested weir, taking into account the streamline inclination and curvature at the brink. A general end-depth-discharge relationship, for both subcritical and supercritical flow was obtained. Discharges obtained from this relationship were compared with experimental data and with those obtained from other theoretical methods. The surface profiles for the zone between the brink and an upstream section with hydrostatic pressure distribution were also investigated.

Ramamurthy et al. (2004) [43] formulated an accurate relationship between end depth and discharge rate for a horizontal trapezoidal free over fall. They included the effects of non-uniform velocity distribution and curved streamline at the free over fall to obtain a relationship between end depth and discharge. The measured static pressure head distribution agreed well with the predicted value for the end section. The pressure force at the end section was obtained from the measured static pressure distribution at that section.

Ramamurthy et al**.** (2006) [44] developed a VOF (Volume of Fluid) model to simulate the flow over a free over fall in trapezoidal channels. The model was used to predict pressure head distribution, velocity distribution and water surface profiles for the over fall. The predicted values were checked using existing experimental data.

Pal and Goel (2007) [33] applied modeling technique (radial based kernel and polynomial kernel) based on support vector machines to determine discharge and end-depth of a free over fall occurring over a smooth trapezoidal channel with horizontal and sloped bottom. The predicted values of both discharge and end depth were compared with previously derived empirical relations and also with a back propagation neural network model. In case of discharge prediction, correlation coefficient was more than 0.995 with all three different slopes, while it was more than 0.996 in predicting the end depth using radial based kernel of support vector machines algorithm. A smaller computational time was an advantage of using support vector machines.

Vatankhah (2013) [54] presented a theoretical end depth–discharge (EDD) relationship for free over fall (end section) in a horizontal trapezoidal shaped open channel. Two direct discharge equations in terms of end depth for subcritical flow were proposed by simulating free overfall as a weir without crest. The calculated discharges, using the proposed EDD relationships agreed well with the experimental data.

2.3 CIRCULAR CHANNEL

Replogle (1962) [37] carried out his investigation for circular channel based on several assumptions used in Diskin's momentum equation (1961). He developed similar momentum equations and showed that the effect of energy correction factor (α), momentum correction factor (β) and residual pressure is small. However they may account for approximately 5% difference of actual discharge and discharge given by Diskin momentum equation.

Smith (1962) [46] studied end depth problem in circular channels by assuming unit momentum coefficient and zero pressure at the end section to get an exact solution for the flow area (A_d) at a vertical section beyond the end. From his experiments, he found actual area was always greater than computed area. He also gave a dimensionless curve giving the variation of (D_e/D) with ($Q/D^{5/2}$) for calculation of discharge for a freely discharging circular pipe of any size with upper limit $Q/D^{5/2}$ as 3.7.

Diskin (1963) [8] proposed an equation for circular channels having a logarithm relationship between $Q/(gD^5)^{1/2}$ and Y_e/D as

$$Q/(gD^5)^{1/2} = 1.82\,(Y_e/D)^{1.96} \quad (13)$$

The values obtained from the above equation vary 0.5% from the value obtained from momentum equation for the range $0.05 < Y_e/D < 0.75$.

Rajaratnam and Muralidhar (1964) [39] investigated on end depth in circular channel based on momentum approach. Based on experimental data they gave an equation for discharge in terms of end depth as

$$Q/(gD^5)^{1/2} = 1.54\,(Y_e^{1.84}/D) \quad (14)$$

When the channel was sloping Y_e / Y_C was found to be a function of S_o / S_C. The value of Y_e/Y_C decreased from 0.75 to 0.487 as S_o/S_C was increased from -4.0 to 8.0.

Subramanya and Niraj Kumar (1993) [50] gave an expression for end depth at free over fall of a horizontal circular channel based on energy consideration and continuity of water surface profile at the end section. The major expression given by him for horizontal frictionless circular channel having sub critical flow was

$$6\,F(Y_C/D) - 4\eta - 3\,f(\eta, Y_C/D) = 0 \quad (15)$$

Where $F(Y_C/D) = 1.0 + 0.0625[(2\theta_C - \sin\theta_C)/(\sin\theta_C * (Y_C/D))]$

And, $f(\eta, Y_C/D) = [0.125\,(2\theta_C - \sin 2\theta_C)^3] / [(Y_C/D)(2\theta_e - \sin 2\theta_e)^2 \sin\theta_C]$

In the above equation, θ_C and θ_e are the angles made by Y_e and Y_c at the center of the circle. The equation was solved by computer and results showed that variation of Y_e/Y_C was relatively small and was taken as constant 0.730. For calculating the discharge for a given end depth, a calibration curve was plotted between $Q/(gD^5)^{1/2}$ *and* Y_e/D based on available experimental data of both smooth and rough channels.

Tiwari (1994) [52] developed an expression of free over fall at circular channel by applying momentum approach and developed computer software. Effect of weight of control volume on sloping floor was included in the derivation. For circular channel (diameter D and central angle 2θ radians), the equation developed by him showed that end depth ratio for the channel was a function of θ_C which in turn is a function of Y_C/D. For sloping channels, EDR was found to be a function of relative slope.

Dey (1998) [9] theoretically analyzed to calculate end depth ratio for smooth circular channel based on momentum approach for both sub critical and super critical flow. For sub critical flow, end depth ratio was found to be 0.75 for critical depth diameter ratio up to 0.82. For super critical flow, end depth was expressed as function of slope of the channel by using Manning's formula. Expression for discharge was also proposed for both sub critical and super critical flow. He also determined the upstream flow profile and an auto recursive search scheme to analyze the free over fall in horizontal circular channel.

Dey (2001) [10] derived a simplified approach to determine end depth of a free over fall in horizontal and mildly sloping circular channel. The EDR for a circular channel was obtained by simulating the flow by that over a sharp crested weir by making coefficient of velocity as a free parameter. The theoretical model was compared with existing experimental data. The EDR varies almost linearly 0.72 to 0.74 for a critical depth diameter ratio up to 0.86. He obtained an expression for discharge. The model was well accordingly with the experimental data.

Nabavi et al. (2009) [32] gave a theoretical model to measure the flow by end depth method for horizontal or mildly sloping inverted semicircular channels. They analyzed based on momentum approach to give expression for EDR, whose value was found to be 0.7 for critical depth-diameter ratio up to 0.4. They also gave an expression for discharge. The theoretically obtained values agreed well with the experimental data of Subhasish Dey.

Sharifi et al. (2011) [51] used genetic programming (GP) for modeling free over fall of circular channels to obtain an expression for end depth ratio. By applying GP to experimental data of circular channels with a flat bed and employing a model selection procedure, they derived the expression as

$$Y_C/\ Y_e = Ae^{B\sqrt{S}} \qquad (16)$$

It was used for calculating the critical depth (Yc) and end-depth ratio (EDR). This expression was dimensionally correct (unlike some other applications of GP) and can be used for channels with any cross-section and any flow regime.

Dey (2003) [13] carried out both experimental and theoretical study on free over fall of a smooth inverted semicircular channel. Based on momentum approach, he found the expression for the end depth ratio, which eliminated the need of an empirical pressure coefficient. For sub critical flow, EDR was found to be 0.705 for a critical depth diameter ratio up to 0.42 while for super critical flow the end depth was expressed as function of channel slope by applying Manning's equation. Discharge was also estimated for both subcritical and supercritical flow and related with end depth and other characteristic parameter and upstream surface profile was computed. He carried out experiments in three inverted semicircular channels made of transparent Perspex, having diameter of 128mm, 68mm and 43mm and length of 4m. The computed values agreed well with experimental data except for supercritical flow which showed a little variation.

Rashwan and Idress (2013) [45] carried out an experimental and mathematical study to evaluate efficiency of brink as discharge measurement device in horizontal, mild and partially filled circular open channels. Mathematical equation was derived on the basis of momentum, discharge and Froude number expressions. The proposed model was calibrated with experimental data. Discharge was accurately calculated from end depth. Results of the laboratory experiments agreed with the calculated values and showed that circular flumes can be effectively used to measure low flow rates in open channels.

2.4 TRAINGULAR CHANNEL

Replogle (1962) [37] carried out his investigation for rectangular, triangular and circular channels based on several assumptions used in Diskin's momentum equation (1961).). He developed similar momentum equations and showed that the effect of energy correction factor (α), momentum correction factor (β) and residual pressure is small, however they may account for approximately 5% of difference between actual discharge and discharge given by Diskin momentum equation. He also measured the pressure and velocity distribution for the channels.

Rajaratnam and Muralidhar (1964) [38] investigated on end depth at free over fall in exponential channels based on momentum approach. By solving the main equation, they obtained general expression of end depth at free over fall for rectangular (n=1), triangular (n=2) and parabolic (n=1.5) channels. The experimental channels had smooth surface. For triangular channel, K_1 and Ye/Y_C was found to be 0.1 and 0.795 respectively. For sloping channel, end depth ratio was found to be a function of relative slope.

Tiwari (1994) [52] developed an expression of free over fall at triangular channel by applying momentum approach and developed computer software for its solution. Effect of weight of control volume on sloping floor was included in the derivation. For triangular channel (side slope 1 in m), the equation developed by him was

$$K_1\ (Y_e/Y_C)^5 - (5/2)\ (Ye/Y_C)^2 + (3/2) = 0 \qquad (17)$$

The above equation is same as obtained by Rajaratnam et al. For horizontal bottom and zero end pressure, end depth ratio was(16) found to be 0.7746 and it was same as given by Diskin.

Mittal and Desmukh (1998) [26] developed computer software in FORTAN-77 language to estimate the flow in triangular free over fall. They prepared calibration curves for triangular channel, with the help of which discharge can be determined if brink depth is known. For triangular free over fall, equation of curve obtained by applying least square curve fitting technique was,

$$Y_R = 0.777 + 0.038K + 0.106K^2 \qquad (18.1)$$

$$Q^* = 0.753 + 0.065K + 0.325K^2 \qquad (18.2)$$

Q^* is the non dimensional discharge and Y_R is the end depth ratio. Another expression derived for triangular over fall curve was $Q=1000\ [gY_b^5]^{0.5}\tan\theta\ Q^*$. In this paper, they had taken angle for triangular over fall as 45^0. The error involved by using these equations was within 1% except for some higher values of K.

Nabavi (2008) [31] computed end depth ratio at free over fall of triangular channels by applying momentum equation. He gave a theoretical model to predict the pressure head distribution at the brink of free over falls, in a smooth △-shaped (equilateral triangle-shaped) channel. In sub-critical flows, the EDR related to the critical depth was found to be 0.695 for critical depth-channel height ratio up to 0.6. In super-critical flows, the Manning equation was used to express the end-depth as a function of the upstream Froude number and relative bottom slope (S_0/Sc) of the channel. Discharge was estimated from the end-depth in sub-critical and super-critical flows. The discharge was also related to the end-depth and a characteristic parameter of the channel. The results obtained agreed well with the results of Dey for subcritical flow.

2.5 U SHAPED

Tiwari (1994) [52] derived an expression for end depth ratio at free over fall of a U shaped channel (equation of channel $Y = BX^2$, parabolic) by applying momentum approach and also developed computer software for this propose. For horizontal bed condition of the channel, equation developed was

$$K_1 (Y_e/Y_C)^4 - (8/3)(Ye/Y_C)^{3/2} + (5/3) = 0 \quad (19)$$

The equation was same as that obtained by Rajaratnam et al. For $K_1 = 0$, the above expression yielded to $Y_e / Y_C = 0.731$, which was same as that obtained by Diskin.

Dey (2005) [11] theoretically analyzed the free over fall in horizontal U shaped channels based on momentum equation to obtain an expression for end depth. The experiments were conducted in two 4m long U shaped horizontal channels having width of 130mm and 70mm made of transparent Perspex sheet. The height of channels was three times of width. The EDR was found to be 0.75 up to non dimensional critical depth 0.5 and then it increases with increase in non dimensional critical depth. Estimation of discharge was also done from mathematical solution of end depth. Stream line curvature at free surface was used to obtain an expression for free surface profile. The results obtained from theoretical analysis well agreed with experimental data. The method eliminated the need of an empirical pressure coefficient.

3. CONCLUSION

Flow measurement of any open channel flow is a vital aspect of its design and levy charged by the users. End depth or brink depth is a simple way for estimation of discharge in all shapes of channels. Till now it has always taken an attention by various researchers and a number of works have carried out in this field. During the literature review, it was found that a lot of work has been carried out till date on various shapes of open channels like rectangular, trapezoidal, circular, triangular, parabolic, U shaped etc. for both horizontal and sloping channels.

i. In case of rectangular channel, many experimental woks have been carried out to determine a particular relationship between end depth and discharge for sub critical and super critical flow and for both smooth and rough channels. But still there is a scope to examine the rectangular channel having sudden contraction and expansion along its length.

ii. In case of trapezoidal channel, number of experimental and theoretical works has been carried out for smooth channels. There is a need to analyze the effect of roughness on both trapezoidal channels.

iii. A lot of analytical and experimental studies have been carried out in circular channel free over fall. But still there is a scope to determine the effect of roughness on horizontal and sloping circular channel. Discharge at free over fall of a circular channel should be analyzed for pressurized or transitional flow condition.

iv. Many theoretical model and experimental works have been carried out in triangular and U shaped channels for smooth channel. But the effect of roughness on adversely sloping channel shape need to be considered at the over falls.

In all it required to examine effect of roughness on the behavior of brink depth, there by the estimation of discharge.

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