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Effect of Vertical Vibration on Block Foundation Resting on Homogeneous and Layered Medium

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Abstract
In the present study, an investigation is carried out to determine the effect of soil–rock and rock–rock foundation systems on dynamic response of block foundations under vertical mode of vibration. The half-space theory is used for the analysis of foundation resting on homogeneous soil and rocks. The finite element program having transmitting boundaries is considered for layered system considering soil–rock and rock–rock combinations. The analysis is carried out in details for soil–rock and weathered rock–rock systems and the different equations are presented for above combinations. The effect of top layer thicknesses, shear wave velocity and eccentric moments are also simulated. The rock–rock systems considered are sandstone, shale and limestone underlain by basalt rock. It is interpreted that as the shear wave velocity ratio increase the natural frequency increases and the peak displacement amplitude decreases.

Keywords:- Block foundation, Vertical vibration, Soil–rock, Rock–rock,

I. Introduction

The geology of earth surface is very vast therefore, it is very rare to get homogeneous soil in natural state and it can exist in a state with a hard rock at shallow depth, consisting of different strata with different properties. The two layered systems which are commonly available in nature are: (i) soil-rock system and (ii) rock-rock system. The dynamic behavior of machine foundation placed on non-homogeneous rocks is very complex due to bedding foliation plane with varying strength, fissures, joints, and faults. The dynamic force on machine foundation is mainly due to operation of rotating type machinery. Due to complexity of the problem in machine induced vibrations on soil-rock and rock-rock system there is a need of research addressing these issues.

The basic mathematical model used for the dynamic analysis of machine foundation resting on homogeneous medium is a mass-spring-dashpot model. This model is used by many researchers [1, 2] to study the dynamic response of footing. Many researchers have used an analytical solution which considers the foundation soil system as an oscillating body resting on semi-infinite, homogeneous, isotropic, elastic half-space. Reissner [3] developed an analytical solution by using elastic half-space mathematical model for periodic vertical displacement at the center of the circular loaded area. Sung [4] considered the effects of the contact pressure due to the vertical vibration. Lysmer and Kuhlemyer [5] and Veletsos and Wei [6] introduced finite element and boundary element solutions.

Many researchers found that natural frequency of the foundation soil system depends on shape and size of the foundation, depth of embedment, dynamic soil properties, nonhomogeneities in the soil, frequency of vibration etc. The effect of footing resting on layered soil was studied by some researchers [7, 8, 9]. Baidya [10] developed a solution to determine the stiffness of the *i*-th layer of a multilayered system and the applicability of this theory have been verified by Baidya and Rathi [11] and Baidya et al. [12] from model block vibration test results. However, no studies are available in the literature to determine the stiffness and damping of foundation resting on various soil-rock and rock-rock system.

In the present paper, an attempt is made to investigate the frequency amplitude response of block foundation in layered soil-rock and rock-rock system. The size of machine foundation block considered for the present study is 0.75×0.75×1.0 m (as per IS 5249). The non-homogeneity incorporated in the present study is due to the horizontal bedding. In the analysis half-space theory is used for homogeneous medium (rock or soil). The system is assumed to be composite medium for layered combination. The results obtained from the analysis of homogeneous medium and rock-rock system shows the variation of normalized stiffness, damping and amplitude with dimensionless frequency. The rock-rock combinations considered in present study are sandstone, shale and lime stone underlain by basalt. The graphs are plotted to show the variation of dimensionless natural frequency and resonant amplitude with shear wave velocity ratio for soil-rock and weathered rock-rock systems.

II. Theoretical Study

1. Homogeneous medium

The frequency dependent stiffness and damping of foundation resting on the homogeneous strata as halfspace is determined by the theory proposed by Veletsos and Verbic [13]. An approximate solution is presented for the steady state response of rigid disk with mass which is supported at the surface of a viscoelastic halfspace and is excited by a harmonically varying vertical force (= $P_z e^{i\omega t}$, where t = time; ω = circular frequency of the excitation; and $i = \sqrt{-1}$). The response of the system is interpreted in terms of the response of an equivalent spring-dashpot oscillator with mass.The dynamic characteristic for vertical vibration system can be specified by its circular natural frequency (p_z) and by the percent of critical damping (ξ_z). These quantities can be computed as

$$p_z = \sqrt{\left(\frac{B_z}{B_z + B_z^*}\right)}\left(p_z\right)_o \tag{1}$$

$$\xi_z = \frac{C_z^*}{2\sqrt{\left[K_z\left(m+M_z^*\right)\right]}} \tag{2}$$

where, B_z = dimensionless inertia ratio; B_z^*= dimensionless parameter; $(p_z)_o$= circular natural frequency of the foundation considering the halfspace as massless; C_z^*= damping coefficient of the equivalent oscillator; K_z = static stiffness of the disk-halfspace system; m = mass of foundation; and M_z^*= mass of the equivalent oscillator.

The displacement amplitude of vertically vibrating system can be computed as

$$u_z = \frac{(u_z)_o}{\sqrt{\left[\left\{1-\left(\omega/p_z\right)^2\right\}^2 + 4\xi_z^2\left(\omega/p_z\right)^2\right]}} \tag{3}$$

Where, $(u_z)_o$ = the static displacement produced by the force amplitude P_z.

2. Layered medium

For the analysis of layered medium, top layer is treated as stratum and bottom layer is considered as halfspace. The dynamic response of layered system is analyzed by using finite element program with transmitting boundaries proposed by Kausel and Ushijima [14].

The FEM program used for analysis is based on the formulation and computer program developed by Kausel [15]. A three-dimensional axisymmetric finite element model with transmitting boundaries was used to model a rigid, embedded circular foundation welded to a homogeneous soil deposit of finite depth resting upon a much stiffer rock-like material.

The vertical static stiffness can be derived as

$$k_v = \frac{4GR}{1-\nu} \tag{4}$$

Static stiffness can be expressed in terms of embedment depth and strata depth as

$$k_v\left(\frac{R}{H},\frac{E}{R}\right) = \frac{4GR}{1-\nu}\, f\left(\frac{R}{H}\right).g\left(\frac{E}{R}\right).h\left(\frac{E}{H}\right) \tag{5}$$

where, f, g, h = Functions such that $f(0) = g(0) = h(0) = 1$, R/H = Depth ratio, E/R = embedment ratio.

The expression for stiffness factor 'f' can be derived as

$$f\left(\frac{R}{H}\right) = 1+1.28\frac{R}{H} \tag{6}$$

Function for embedment effect can be determined as

$$g\left(\frac{E}{R}\right) = 1+0.47\frac{E}{R} \tag{7}$$

Approximate equation for function 'h' can be obtained as

$$h\left(\frac{E}{H}\right) = 1+\left(0.85-0.28\frac{E}{R}\right)\frac{E/H}{1-E/H} \tag{8}$$

The empirical expression given by Kausel and Ushijima [14] for the static vertical stiffness using Eqs. 5, 6, 7 and 8 as follows:

$$K_V = \frac{4GR}{1-\upsilon}\left(1+1.28\frac{R}{H}\right).\left(1+0.47\frac{E}{R}\right).\left(1+\left(0.85-0.28\frac{E}{R}\right)\frac{E/H}{1-E/H}\right) \tag{9}$$

where, R = radius of footing, H = depth of soil layer below footing, E = embedment depth,
G = shear modulus, and ν = Poisson ratio.

The equation for dynamic stiffness coefficient can be derived as

$$k = k_v\left(k'+ia_vc'\right)\left(1+2i\beta\right) \tag{10}$$

where, k_v = static stiffness, β = material damping, a_v = dimensionless amplitude and k',c' = dimensionless function referred to as stiffness coefficients.

The computer program DYNA 5 which is formulated by Novak et al. [16] is used for the present study. This program is used to present the dynamic behaviour of block foundation as frequency response curves for vertical displacement, stiffness, and damping constants for both homogeneous and layered system.

III. Results and Discussions

1. Homogeneous medium

The analysis is carried out for homogeneous soil and rocks by using the theory proposed by Veletsos and Verbic [13], for eccentric moments ($m_e e$, where m_e = mass of eccentric rotating part in oscillator and e = eccentricity of rotating part of oscillator) of 0.278, 0.366 and 0.450 N-m. Four different rocks (sandstone, shale, limestone, basalt) are considered for analysis. The results so obtained are then plotted in dimensionless form.

Dimensionless amplitude

$$x_o = \frac{u_z.m}{m_e e} \tag{11}$$

Dimensionless frequency

$$a_o = \frac{\omega R_o}{v_s} \tag{12}$$

where, u_z = amplitude of system; v_s = shear wave velocity; ω = angular operating frequency; R_o = radius of foundation

To normalize the stiffness and damping constant, the stiffness and damping values are divided by stiffness of soil at zero dimensionless frequency ($a_o = 0$)

$$k_o = \frac{k}{k_{Soil}\left(a_o = 0\right)} \tag{13}$$

$$c_o = \frac{c}{k_{Soil}\left(a_o = 0\right)} \tag{14}$$

Where k_o = Normalized stiffness constant, c_o = Normalized damping constant, and $k_{Soil}(a_o = 0)$ = Stiffness of soil at $a_o = 0$

The properties of soil and rocks [17] considered for the analysis are shown in Table.1. Figure 1 shows the variation of normalized stiffness with dimensionless frequency for soil and different rocks respectively. It has been observed that the basalt (igneous rock) exhibits high stiffness value as compared to the other sedimentary rocks (sandstone, shale and limestone) and soil. This is because of the high values of density and the shear wave velocity of the basalt as compared to other sedimentary rock.

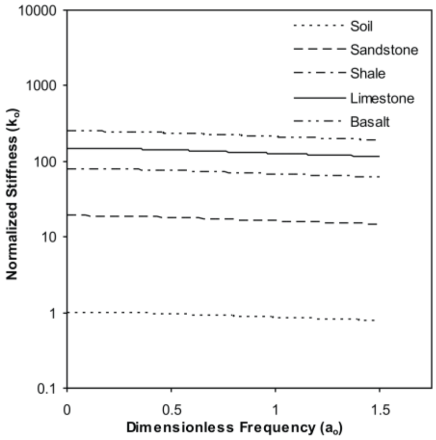

Fig. 1 Variation of normalized stiffness with dimensionless frequency

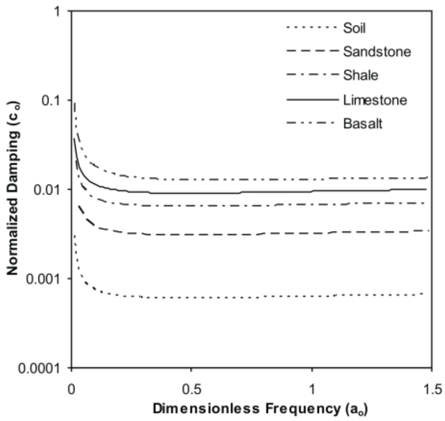

Fig. 2 Variation of normalized damping with dimensionless frequency

Table 1. Properties of soil and rocks

Type of soil and rock	Shear wave velocity (m/s)	Unit weight (N/m^3)
Soil	185	16000
Weathered rock	1680	20810
Sandstone	1110	22000
Shale	2220	23000
Limestone	2960	25000
Basalt	3700	26000

Figure 2 shows the variation of normalized damping with dimensionless frequency for rocks and soil respectively. As the frequency approaches to zero the damping shows an increasing trend. This is because of the conversion of frequency-independent material damping (β) and frequency (ω) to the equivalent viscous damping coefficient (c) as $c = 2\beta / \omega$.

The variation of dimensionless amplitude with dimensionless frequency is shown in Figure 3. It is observed that the dimensionless resonant amplitude of soil is higher as compared to rocks. The dimensionless resonant frequency for soils is less than the rocks. Figure 3 can be used to find the frequency amplitude response of block foundation of any mass and size for homogeneous soils and rocks under vertical vibration.

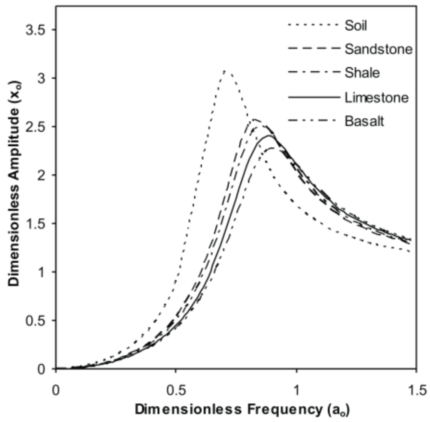

Fig. 3 Variation of dimensionless amplitude with dimensionless frequency

2. Layered medium

The ratio of depth of top layer (*H*) with width of footing (*B*) is taken as (*H/B*) = 1.0, 1.5, and 2.0 to investigate the dependency of frequency amplitude response on depth of top layer. The three different eccentric moment considered in analysis are $m_e e$ = 0.028, 0.037, and 0.045 kg-m. The results are plotted in the form of dimensionless parameters i.e dimensionless frequency a_o, dimensionless amplitude x_o etc. The normalized stiffness(k_o) and damping(c_o) of layered system are obtained by the following equations

$$k_o = \frac{k_l}{k_{Basalt}\left(a_o = 0\right)} \quad (15)$$

$$c_o = \frac{c_l}{k_{Basalt}\left(a_o = 0\right)} \quad (16)$$

Where, k_o and c_o are normalized parameters for stiffness and damping of layered system, k_l and c_l are frequency dependent stiffness and damping constant of layered system, $k_{Basalt}(a_o = 0)$ is the stiffness of basalt rock at the zero dimensionless frequency.

Soil-rock system and weathered rock-rock system

The analyses are performed by considering three different shear wave velocity ratios (0.8, 0.6 and 0.3). The dynamic response of footing are obtained for the above mentioned shear wave velocity ratio and *H/B* ratio. The trend lines are then plotted for the graphs between (i) maximum amplitude and shear wave velocity ratio and (ii) natural frequency and shear wave velocity ratio. The graphs are extended for the lower values of shear wave velocity ratios ($V_{s1}/V_{s2} < 0.5$,where V_{s1} = shear wave velocity of top layer, and V_{s2} = shear wave velocity of half-space) by using the trend line equations, to simulate the different soil-rock combination and weathered rock-rock combination systems.

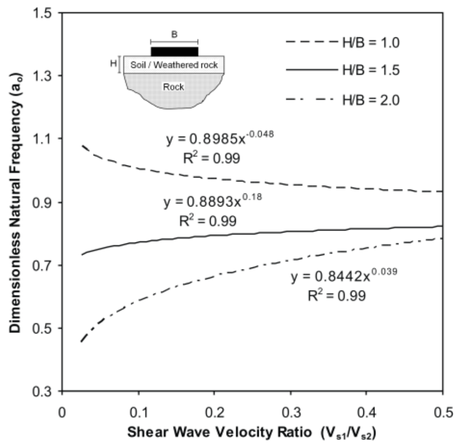

Fig.4 Variation of dimensionless natural frequency with shear wave velocity ratio

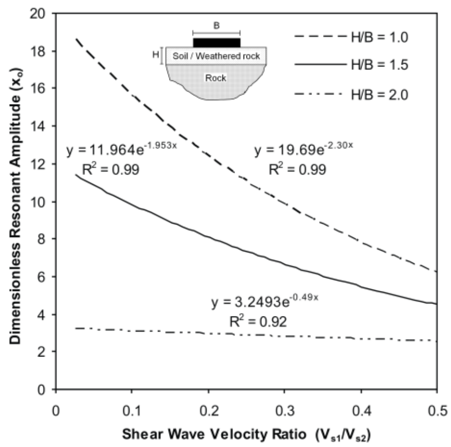

Fig. 5 Variation of dimensionless resonant amplitude with shear wave velocity ratio

Figure 4 shows the variation of dimensionless natural frequency with shear wave velocity ratio. The natural frequency is found to be constant for all vertical force of excitation for the given *H/B* ratio as nonlinearity is not considered in the analyses. It can be seen from the figure that all three trend lines are approaching towards the one constant value, which can be obtained at the shear wave velocity ratio of 1. Figure 5 shows the variation of dimensionless resonant amplitude with shear wave velocity for different *H/B* ratio. The variations of dimensionless resonant amplitude with shear wave velocity for different *H/B* ratio are shown by trend line equations. It is found that with increase in depth of top layer (*H*) for the particular shear wave velocity ratio the dimensionless resonant amplitude value decreases.

Rock-rock system

The present study is focused only on non-homogeneity due to horizontal bedding plane. Therefore the analysis is carried out on the combinations of sedimentary rock underlain by basalt (igneous rock) which is most common in geological aspect. Sedimentary rocks considered in this study are the sandstone, shale, and limestone.

The variation of normalized stiffness and damping with dimensionless frequency (a_o) for different *H/B* ratios and for different rock-rock combinations are shown in Figures 6 and 7 (sandstone underlain by basalt rock, shale underlain by basalt rock and limestone underlain by basalt rock) respectively.

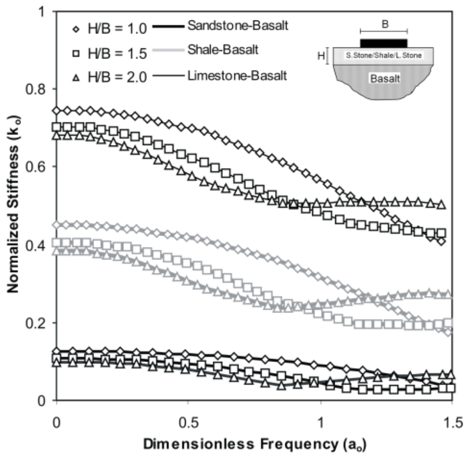

Fig.6 Variation of normalized stiffness with dimensionless frequency (rock-rock)

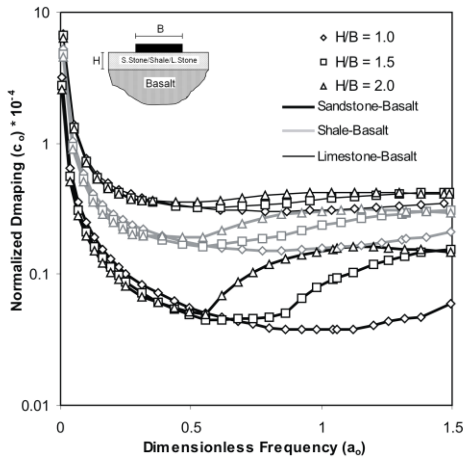

Fig.7 Variation of normalized damping with dimensionless frequency (rock-rock)

It is observed that the normalized stiffness value decreases with dimensionless frequency (a_o). It can be seen that with increase in the depth of top layer the normalized stiffness decreases i.e sedimentary rock layer up to dimensionless frequency value equals to one. It is found from the figures that the stiffness value is less for sedimentary rock-basalt system than the homogeneous basalt rock system. As the frequency approaches to zero the damping shows an increasing trend. The variations of normalized damping with *H/B* ratios are found to be significant for sandstone-basalt combination as compared to the other two systems.

The variations of dimensionless amplitude (x_o) with dimensionless frequency (a_o) are shown in Figures 8 and 9 for different sedimentary-basalt rock combinations. It is observed that with increase in the depth of top layer, i.e sedimentary rock layer, both dimensionless resonant amplitude and dimensionless natural frequency decreases. It can be seen that the value of dimensionless amplitude is maximum for sandstone-basalt system as compared to other two sedimentary rock-basalt systems. The variation of maximum dimensionless amplitude with *H/B* ratios is found to be significant for sandstone-basalt system as compared to the other two systems.

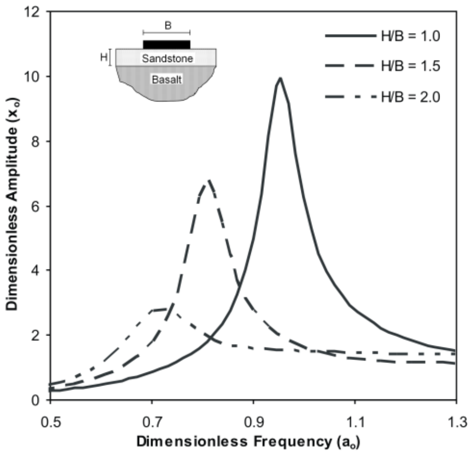

Fig. 8 Variation of dimensionless amplitude with dimensionless frequency for rock-rock system (Sandstone-Basalt)

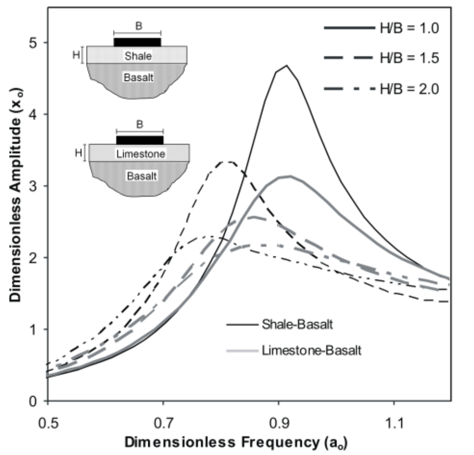

Fig. 9 Variation of dimensionless amplitude with dimensionless frequency for different rock-rock system

IV. Conclusion

In case of block foundation resting on homogeneous systems, it is found that both the normalized damping and normalized stiffness show higher values in case of homogeneous rocks as compared to homogeneous soil. The dimensionless resonant amplitude values are found higher and the dimensionless natural frequency values are found lower for homogeneous soil as compared to homogeneous rock system.

A detailed procedure has been presented for the prediction of dynamic response of block foundation on soil-rock combination and weathered rock-rock combination. The equations proposed for different *H/B* ratios in terms of shear wave velocity ratio can be used to calculate the dimensionless natural frequency and resonant amplitude for soil-rock and weathered rock-rock system. It is concluded that with increase in *H/B* ratio, the dimensionless natural frequency and resonant amplitude decreases. It is also found that the dimensionless resonant amplitude decreases as shear wave velocity ratio increases. In the case of rock-rock combination, it is found that for all *H/B* ratios, the resonant amplitude increases and dimensionless natural frequency decreases with decrease in the shear wave velocity of the top layer. It is observed that the normalized stiffness value decreases with increase in both dimensionless frequency and *H/B* ratio. The variation of normalized damping with *H/B* ratios is found significant for sandstone-basalt system due to the low values of shear wave velocity of top layer.

References

1. Alpan J (1961) Machine foundations and soil resonance. Geotechnique 11(1): 95-113
2. Sridharan A, Nagendra MV (1981) Prediction of frequency and amplitude of foundations at resonance. Can Geotech J 18(4): 603-607
3. Reissner E (1936) Station are axialsymmetricle druch eine elastischen halb raues. Ingenieur Archiv 7(6): 381-396
4. Sung TY (1953) Vibrations in semi-infinite solids due to periodic surface loadings. Symp. on Dynamic Testing of Soils, ASTM, STP No. 156, Philadelphia, pp 35-64
5. Lysmer J, Kuhlemyer RL (1969) Finite dynamic model for infinite media. J Eng Mech Div ASCE 95(EM 4): 859-877
6. Veletsos AS, Wei YT (1971) Lateral and rocking vibrations of footings. J Soil Mech Found Eng Div ASCE 97(SM 9): 1227-1248
7. Kausel E, Roesset JM, Waas G (1975) Dynamic analysis of footings on layered media. J Eng Mech ASCE 101(5): 679-693
8. Gazetas G, Roesset JM (1979) Vertical vibration of machine foundations. J Geotech Eng ASCE 105(12): 1435-1454
9. Kagawa T, Kraft L M (1981) Machine foundations on layered soil deposits. Proc., 10th International Conference on Soil Mechanics and Foundation Engineering, Stockholm, Vol 3, pp 249-252
10. Baidya DK (1992) Dynamic response of footings resting on layered and nonhomogeneous soils. Ph.D. Theiss, IISc, Bangalore, India.
11. Baidya DK, Rathi A (2004) Dynamic response of footing on a sand layer of finite thickness. J Geotech Geoenviron Eng ASCE 130(6): 651-655
12. Baidya DK, Muralikrishna G, Pradhan PK (2006) Investigation of foundation vibrations resting on a layered soil system. J Geotech Geoenviron Eng ASCE 132(1): 116-123
13. Veletsos AS, Verbic B (1973) Vibration of viscoelastic foundations. Earthq Eng Struct Dyn 2: 87-102
14. Kausel E, Ushijima R (1979) Vertical and torsional stiffness of cylindrical footing. Civil Engineering Department Report R79-6, MIT, Cambridge, MA
15. Kausel E (1974) Forced vibration of circular foundation on layered medium, Ph.D. Dissertation, MIT, Cambridge
16. Novak M, El-Naggar MH, Sheta M, El-Hifnawy L, El-Marsafawi H, Ramadan O. DYNA5 - a computer program for calculation of foundation response to dynamic loads. Geotechnical Research Centre, Univ of Western Ontario, London, Ontario, 1999.
17. Saran S (2010), "Soil dynamics and machine foundation", Department of civil engineering, University of Roorkee, Roorkee, India.